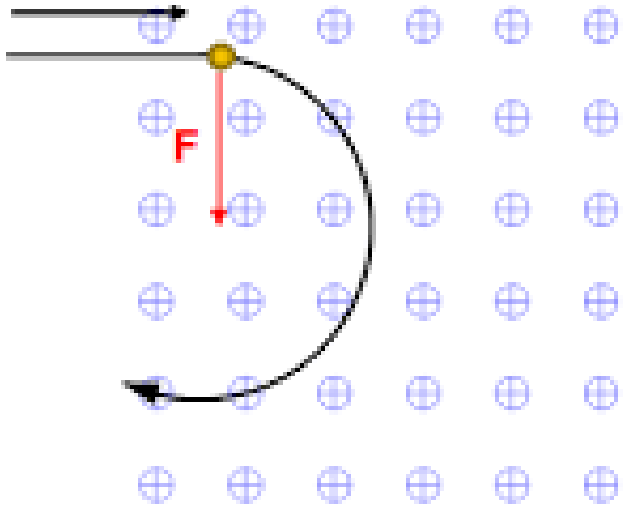
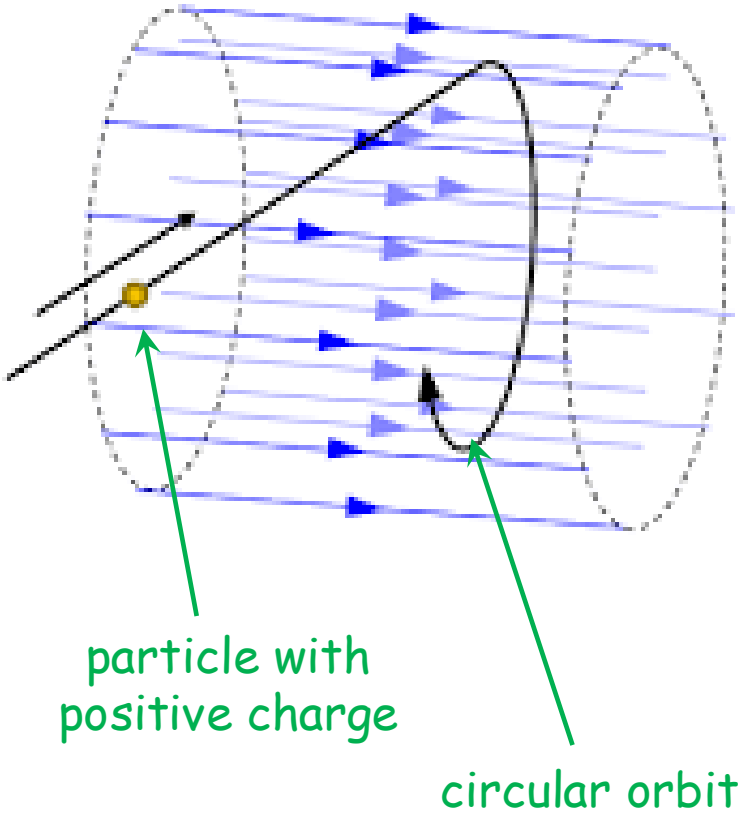
Worlds of Physics (Nov. 8, 2024)

Can A Magnetic Field Do Work?

Philip B. Allen, professor emeritus

A peculiar effect that we teach in freshman physics.







Doing work?

No

Freshman definition:
work = force times distance.



Quiz #1:



Answer #1:

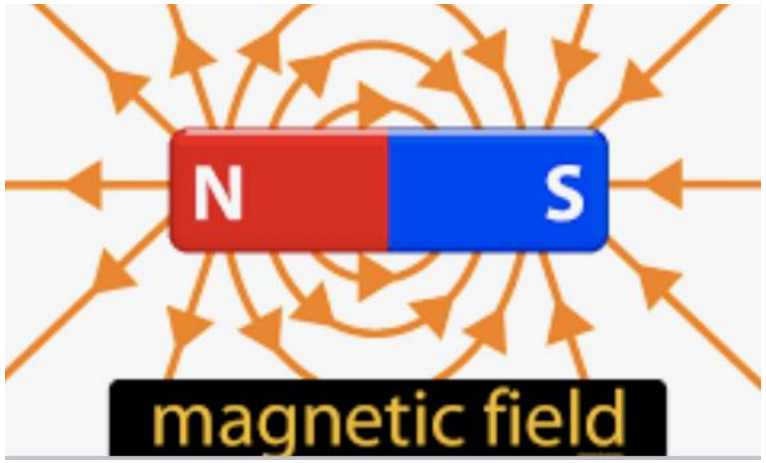


Dictionary

Definitions from Oxford Languages · Learn more

mag·net·ic field

/magˈnedikˈfēld/



noun

a region around a magnetic material or a moving electric charge within which the force of magnetism acts.

Wikipedia

https://en.wikipedia.org › wiki › Field_(physics)

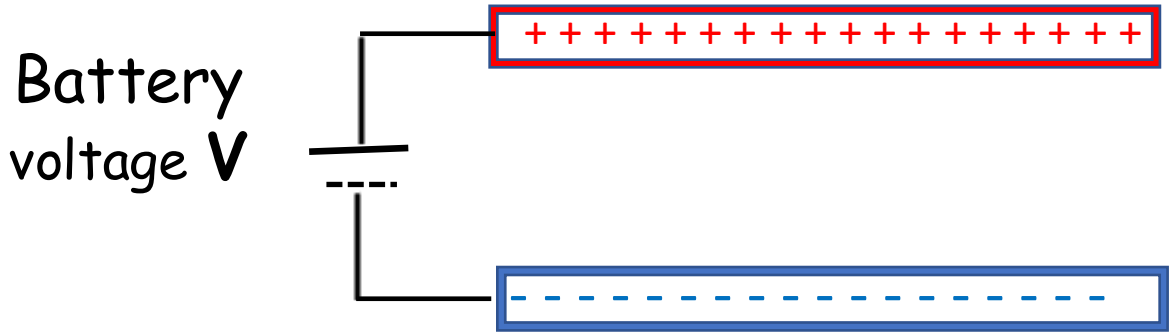
Field (physics)

In science, a field is **a physical quantity, represented by a scalar, vector, or tensor**, that has a value for each point in space and time.

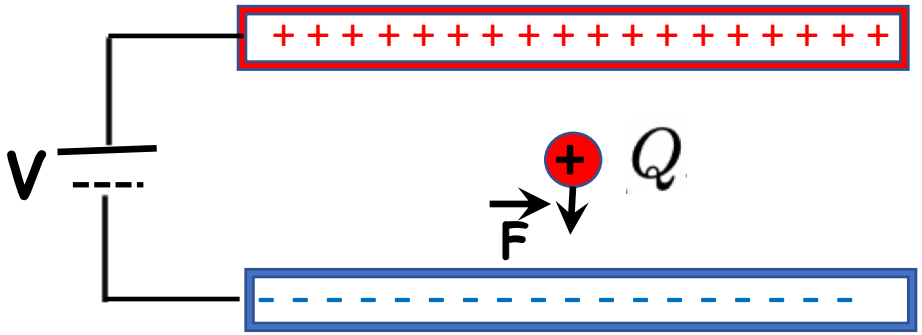
Quiz #2:

Are there other kinds of fields
that physicists talk about?

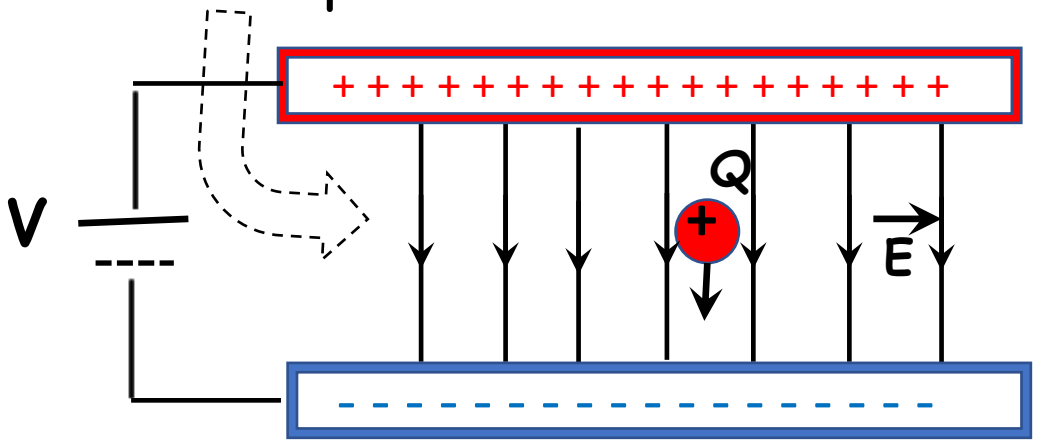
Charged Capacitor – the battery moves negative electrons out of the top plate, into the bottom.



Electric Field & Force



Lines represent the "electric field"



$$\vec{F} = Q\vec{E}$$

charge

Charges on the plates create an "**electric field**" $\vec{E}$

the **field** $\vec{E}$ causes a **force** $\vec{F}$

Dictionary

Definitions from Oxford Languages · Learn more

charge

/CHärj/

noun

1. a price asked for goods or services.
   "an admission charge"

   Similar: fee | price | tariff | amount | sum | figure | fare | rate

2. an accusation, typically one formally made against a prisoner brought to trial.
   "he appeared in court **on a charge of** attempted murder"

   Similar: accusation | allegation | indictment | arraignment | citation | imputation

Wikipedia

https://en.wikipedia.org › wiki › Electric_charge

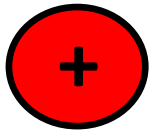
Electric charge

Electric **charge** (symbol q, sometimes Q) is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.

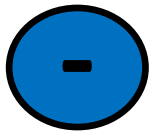
Charged particle

Q

positive charge (example: proton)



negative charge (example: electron)



$Q = \pm e$

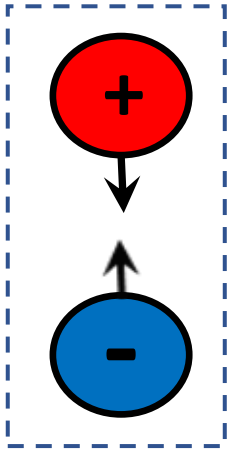
-e is the charge of the electron.

Charge & Force

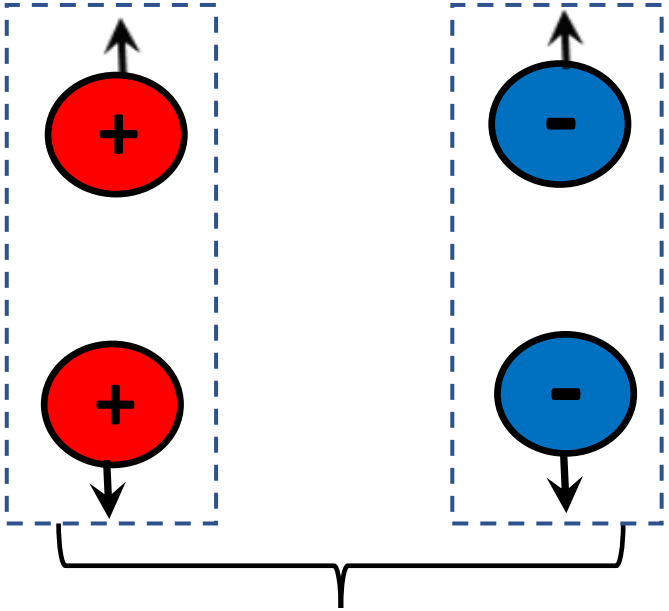
$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}$$

Coulomb's Law

Unit of force
1 pound = 4.45 Newtons



attraction
(opposite charges)



repulsion
(like charges)

Atoms are tiny positively charged nuclei
surrounded by a bigger moving "cloud" of tiny negatively charged electrons.

What other kinds of "fields" do physicists talk about?

Fields that physicists consider to be (actual?) "*things*"

(a) **Electric and Magnetic fields**

(b) Gravitational fields

(c) "weak fields", (d) "strong fields"

An unnecessary digression:

Do we think that the forces are caused by actual REAL fields?

[Not an issue – What does "real" mean?]

Forces are caused by "force carriers":

(a) *photons* (definitely) carry electromagnetic forces

(b) *gravitons* (hypothetically)carry gravitational forces

(c) *W+, W–, and Z bosons* carry weak forces

(d) *gluons* carry strong forces

Dictionary

Definitions from Oxford Languages · Learn more

work

/wərk/

noun

1. activity involving mental or physical effort done in order to achieve a purpose or result.
   "he was tired after a day's work"

   Similar: labor | toil | exertion | effort | slog | drudgery

2. a task or tasks to be undertaken; something a person or thing has to do.
   "they made sure the work was progressing smoothly"

   Similar: tasks | jobs | duties | assignments | commissions | projects | chores

Doing work?
Not doing work?
(some of each)









work (physics)

Written and fact-checked by Encyclopedia Britannica's editorial staff

work, in physics, measure of energy transfer that occurs when an object is moved over a distance by an external force, at least part of which is applied in the direction of the displacement.



"work = force x distance"

amount of work done on an object = increase of the object's energy

Yes! work!



Yes – 2 kinds of work!



The Britannica definition is correct but requires careful application.

"Net work" is done on an object only when the object's **energy** increases.

Yes – The mover is doing positive work, but friction is doing equal negative work.



No! No work if the mover goes at constant speed.
No **"net work"** if the mover starts and later stops.



Dictionary

Definitions from Oxford Languages · Learn more

en·er·gy

/ˈenərjē/

noun

1. the strength and vitality required for sustained physical or mental activity.
   "changes in the levels of vitamins can affect energy and well-being"

   Similar: vitality | vigor | life | liveliness | animation | vivacity | spirit

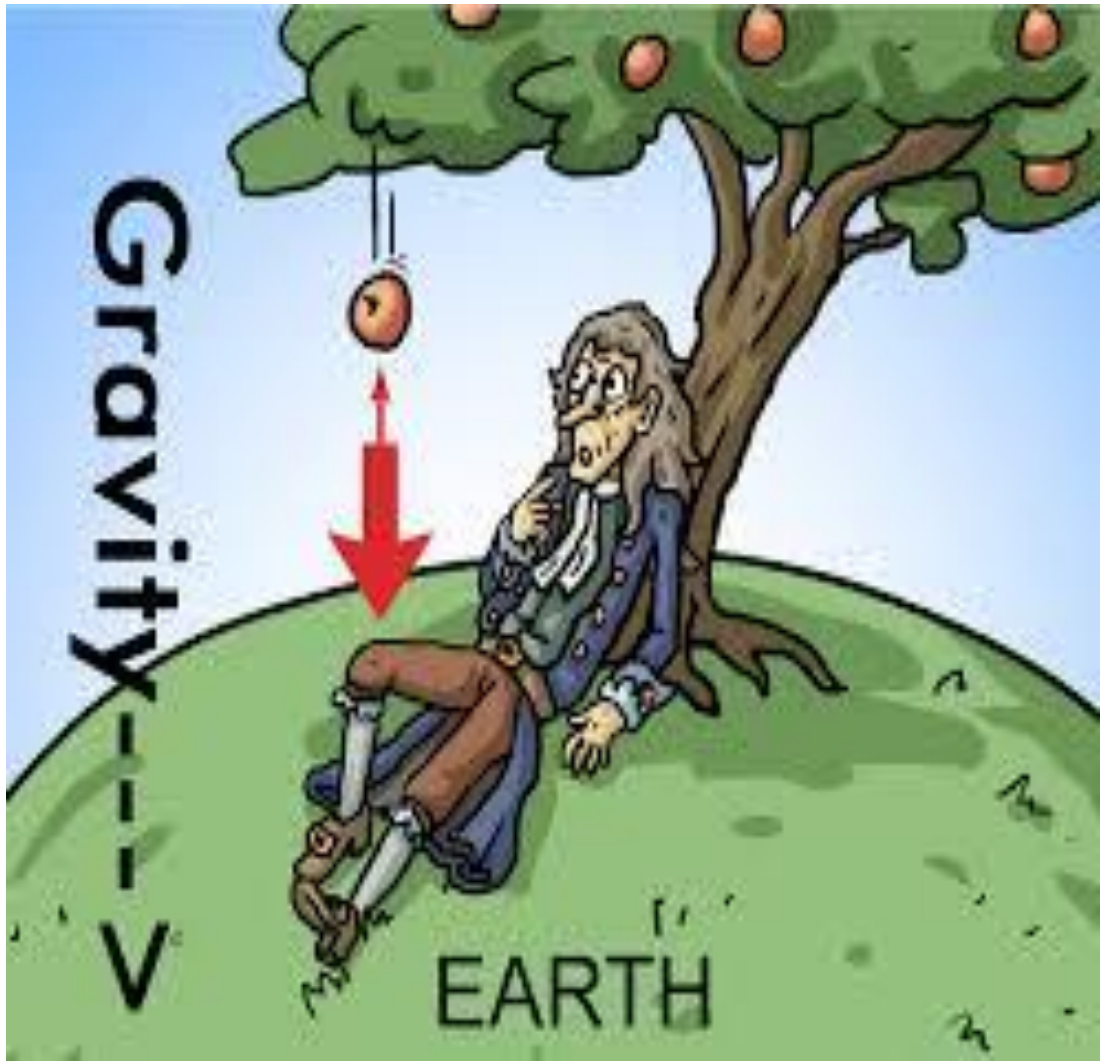
2. power derived from the utilization of physical or chemical resources, especially to provide light and heat or to work machines.
   "nuclear energy"

   Similar: power

The physics definition of energy is similar to 2 above. "Power" in physics is the rate of energy production.


LET US
NOW PAUSE
FOR A MOMENT
OF SCIENCE

Newton's law: F=ma (m=mass, a=acceleration)



Gravity does work
[and increased the apple's "kinetic" energy]

$$\mathrm{KE} = \frac{1}{2}mv^2$$

Force of gravity at earth's surface: **F=mg**
g=acceleration of gravity

W done = $mv^2/2$
= kinetic energy gained

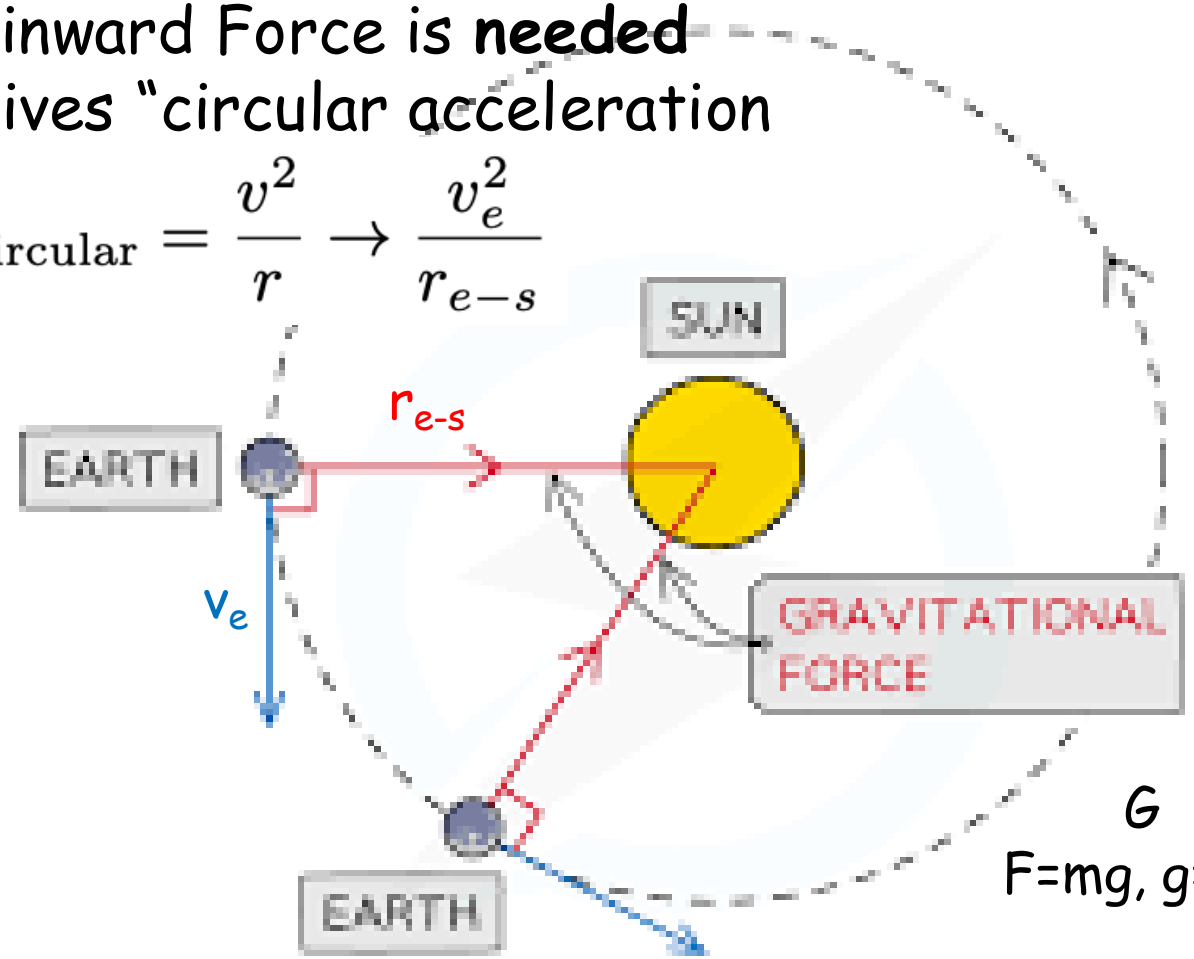
Gravity keeps the earth in it's circular orbit around the sun.

No work is being done! (force x distance = 0).

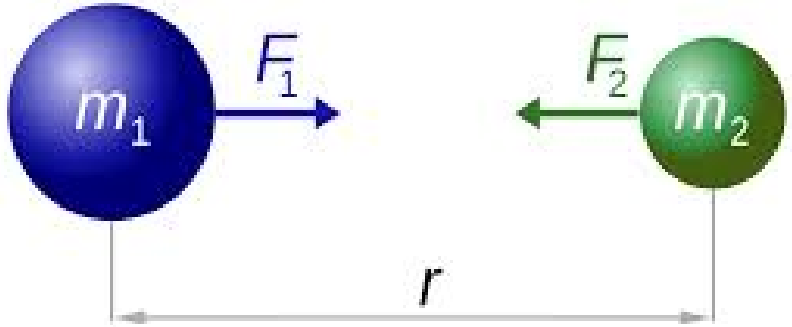
There is **no increase in the earth's kinetic energy** (v is constant)

An inward Force is **needed** and gives "circular acceleration

$$a_{\text{circular}} = \frac{v^2}{r} \rightarrow \frac{v_e^2}{r_{e-s}}$$



Newton's "universal law of gravity"

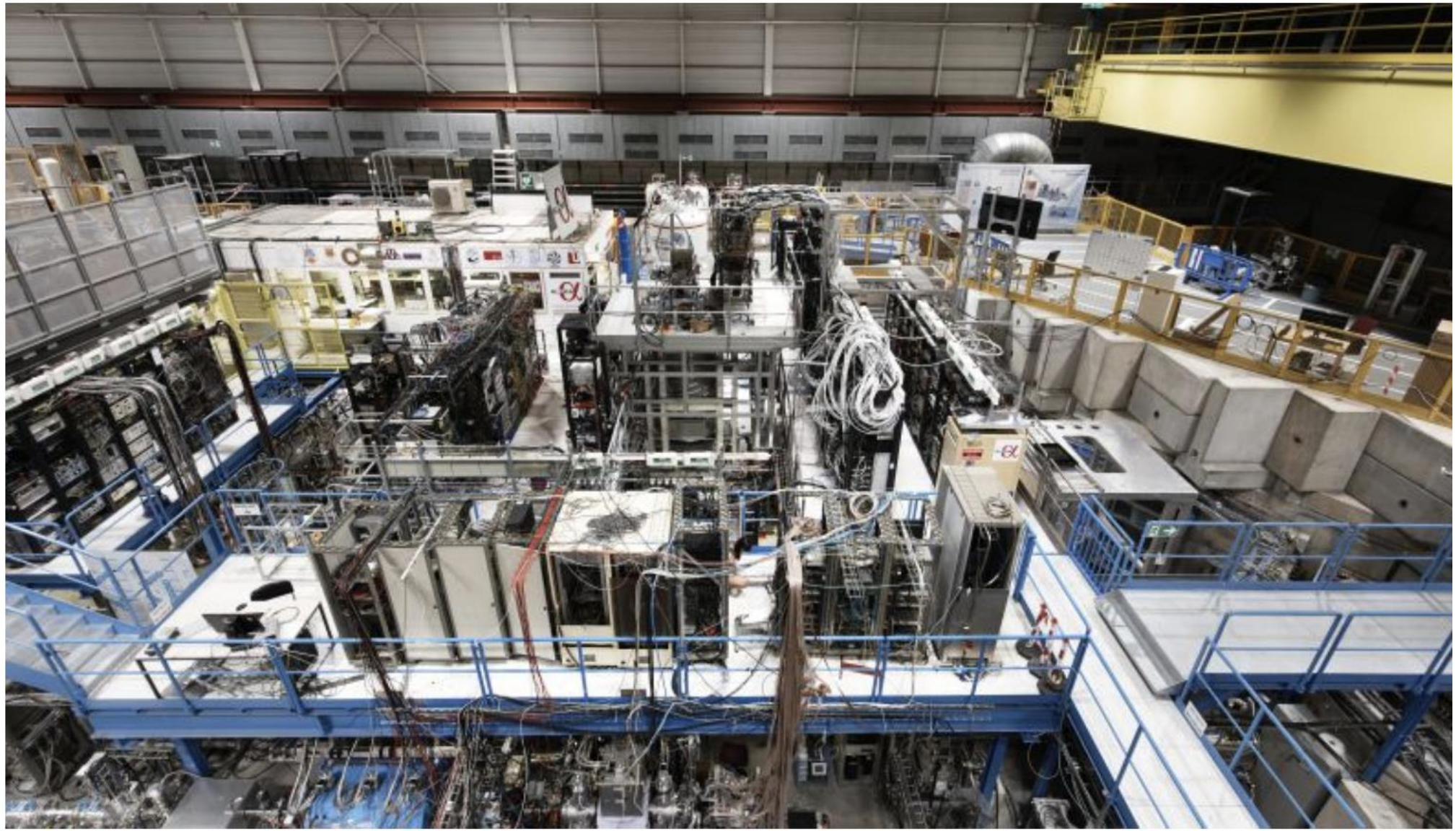


$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

$G = 6.6743 \times 10^{-11}$ Newton meter^2/kilogram^{-2}

$F=mg$, $g=Gm_e/r_e^2=32$ feet/second$^2=9.8$ meters/second^2

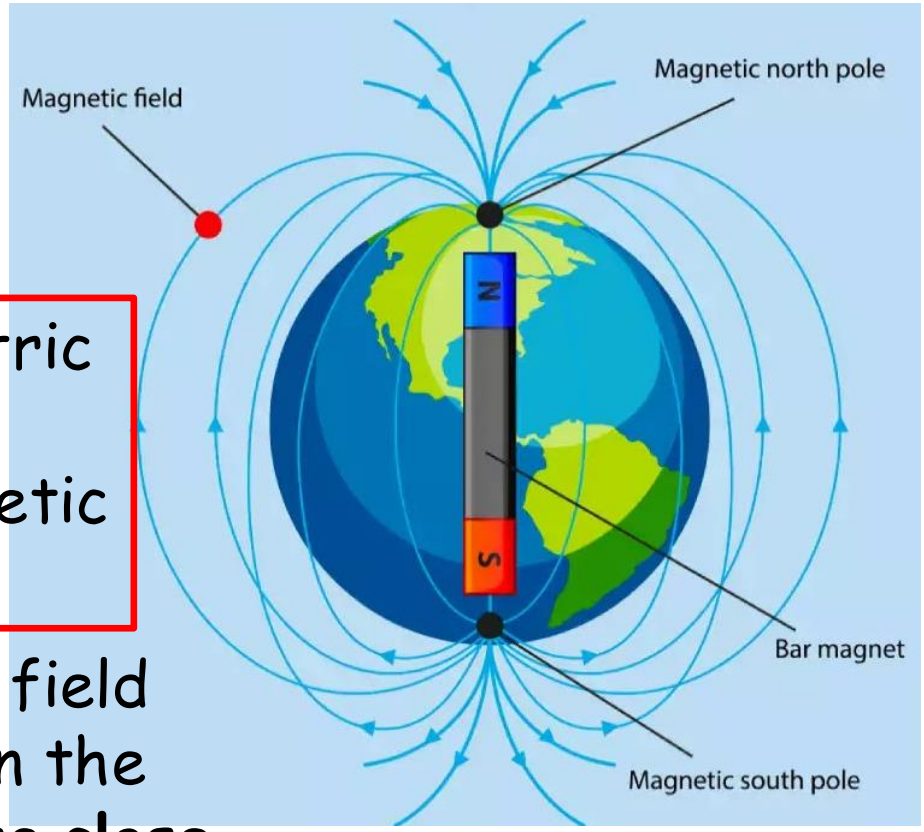
"**Anti-particles**" (anti-proton & anti-electron make an anti-hydrogen atom) are attracted to the earth! [proven experimentally, Sept. 21, 2023.] The mass ***m*** is **never** negative (unlike charge **Q**)..



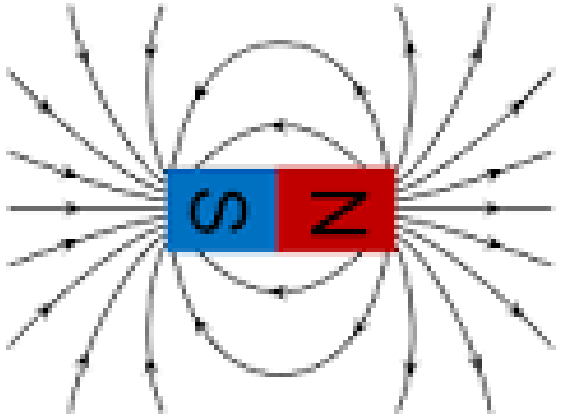
An overhead view of CERN's ALPHA experiment. The cylindrical ALPHA-g apparatus where the antimatter dropping experiment was conducted can be seen under the raised platform at the center.

Magnetic Fields

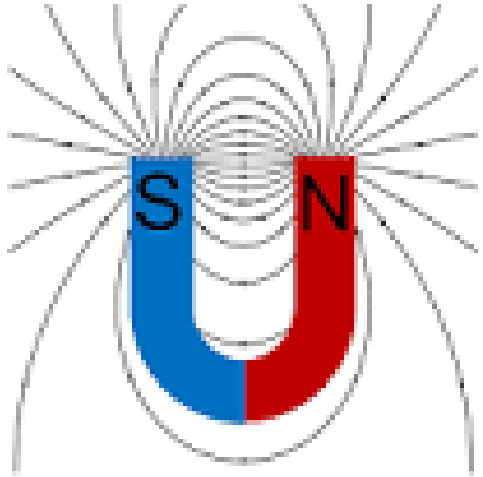
symbol for electric field is **E**
symbol for magnetic field is **B**



The magnetic field is **strong** when the **field lines** (B) are **close**.



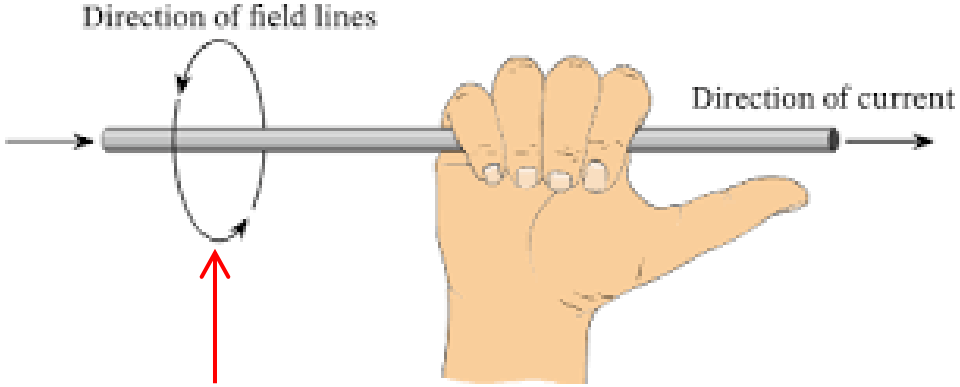
bar magnet



horseshoe magnet

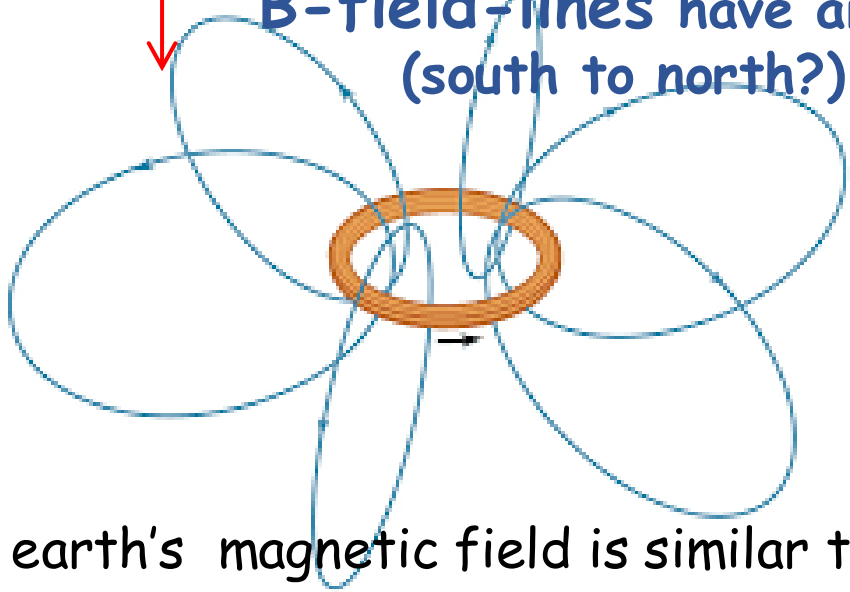
The "fields" are **REAL**.

(The field **lines** are just illustrations.)

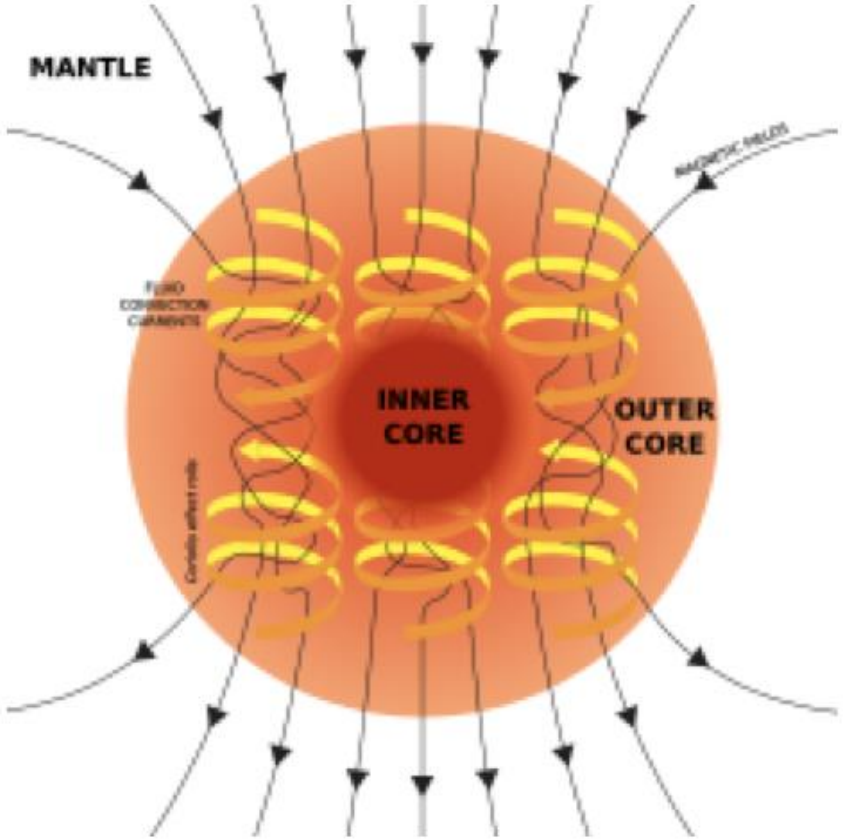


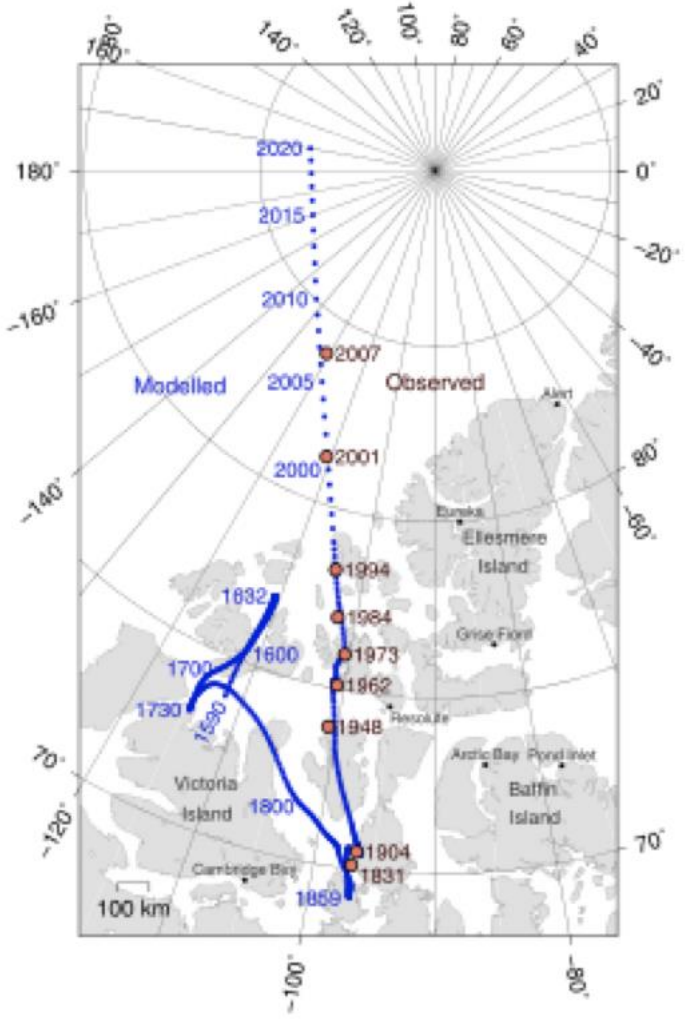
Magnetic field of a current in a wire
Magnetic field of a current in a loop

B-field-lines have arrows (south to north?)



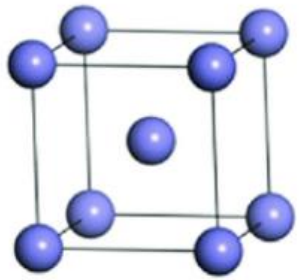
The earth's magnetic field is similar to this.



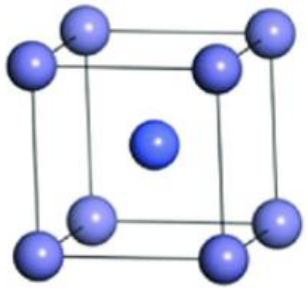


NASA: Our magnetosphere shields us from the solar wind
(charged particles emitted by the Sun),
and cosmic rays from deep space.

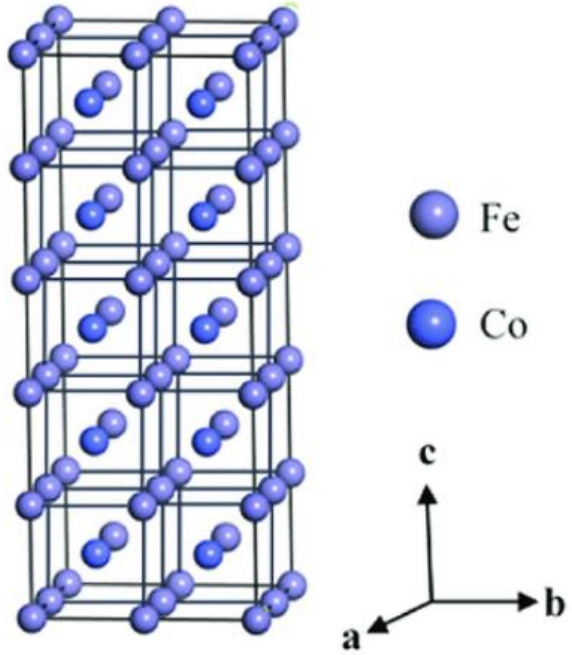
What ARE magnets?



(a) bcc Fe

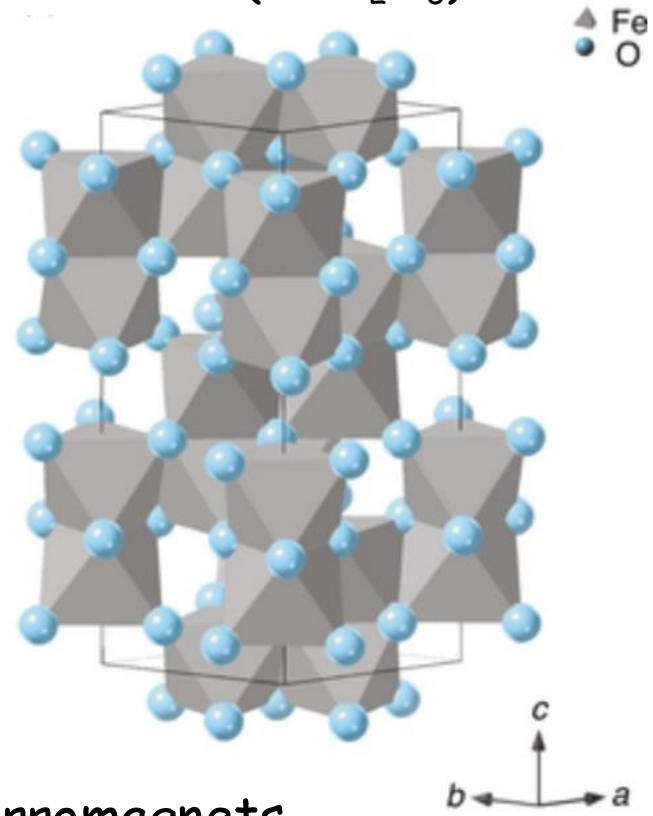


(b) bcc FeCo



(c) Supercell for $Fe_{0.75}Co_{0.25}$

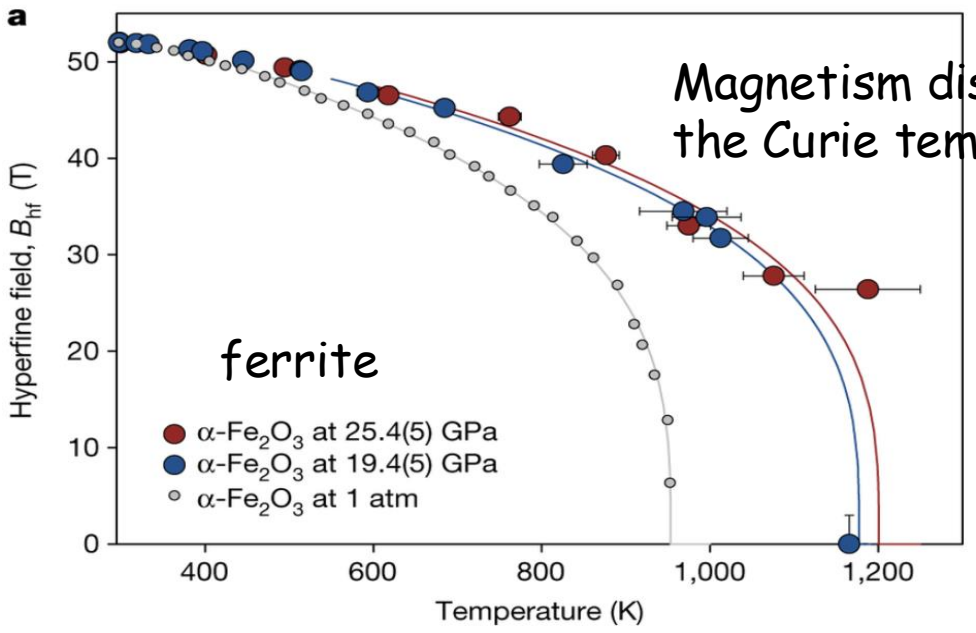
hematite (α-Fe_2O_3)



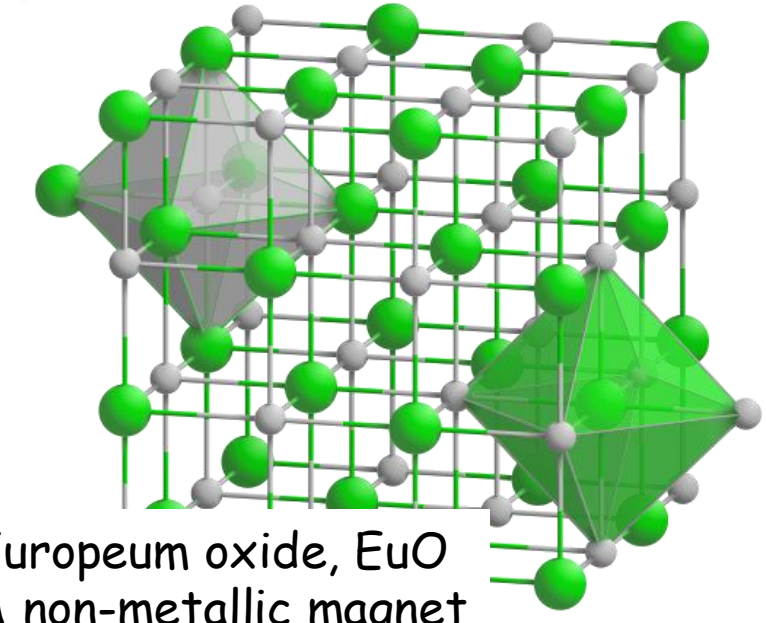
Some metallic elements (Fe, Ni) are ferromagnets.
Some ferromagnetic compounds (EuO) are electrical insulators.
They remain crystalline non-magnetic solids above the "Curie temperature".

Ferromagnets

Material	Curie temperature (K)
Fe	1043
Co	1388
Ni	627
Gd	293
Dy	85
CrBr3	37
Au2MnAl	200
Cu2MnAl	630
Cu2MnIn	500
EuO	77
EuS	16.5
MnAs	318
MnBi	670
GdCl3	2.2
Fe2B	1015
MnB	578



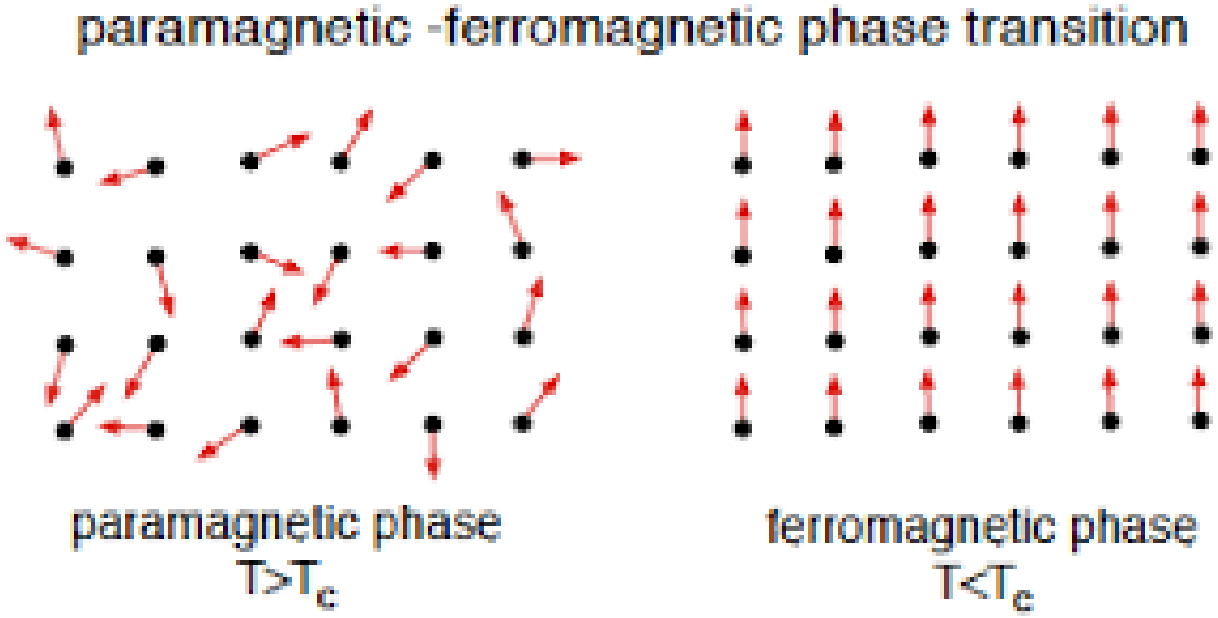
Magnetism disappears above the Curie temperature.



Europeum oxide, EuO
A non-metallic magnet

Magnets are complicated.

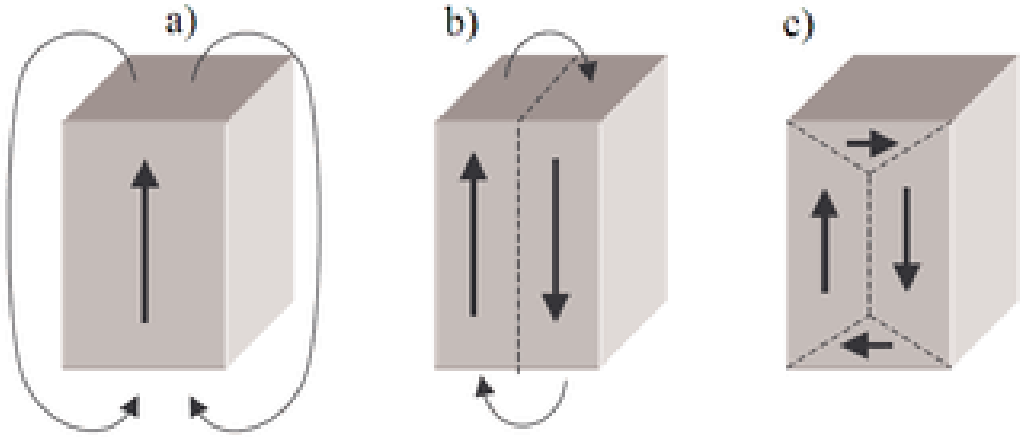
Magnets are crystalline solids made out of atoms. Some special atoms, like Fe, Co, Ni, Mn, Eu, are tiny magnets.



Why is iron not always a magnet?

It prefers to subdivide into "domains".

But it can be coerced into a single domain state, which can be made quite stable. (by lucky alloying)



single domain

Multiple domains

Magnets lifting things.





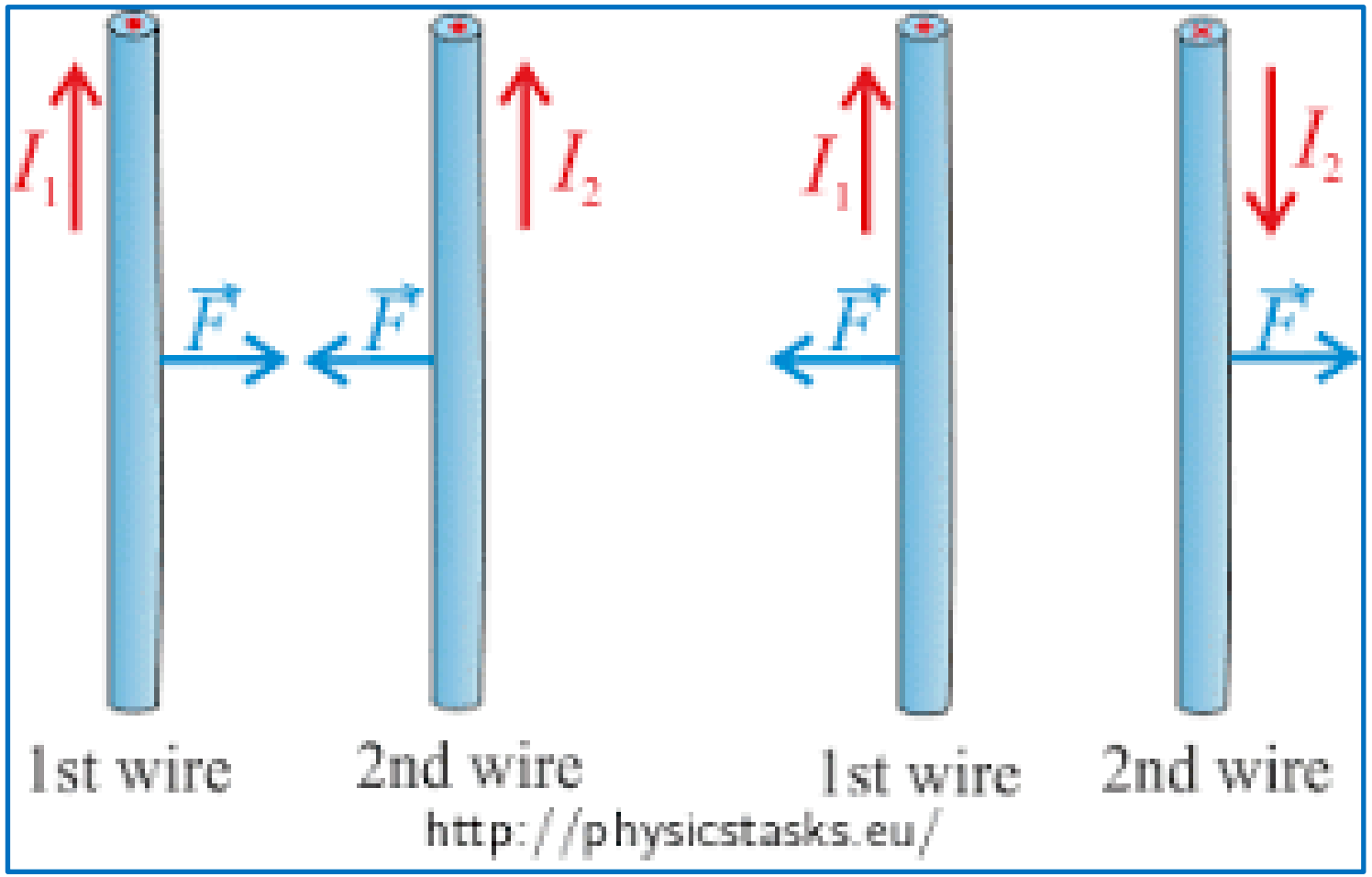


The things getting lifted were not initially in magnetic states. The B-field of the lifting magnet orients domains *inside* the metal being lifted.
The paper clips closest to the magnet got magnetized sufficiently to be able to pick up other paper clips.

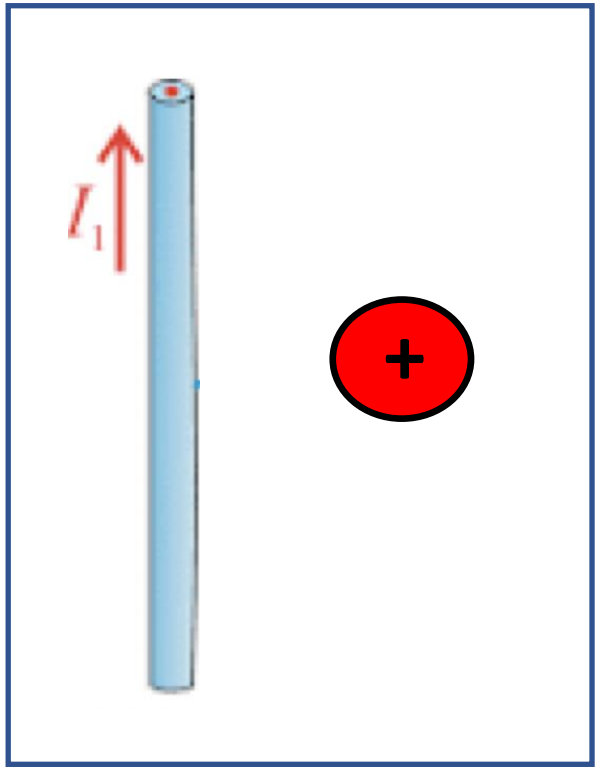
Magnets are very good at doing work!

How can it also be true that magnets do "no" work?

Wires carrying current: Parallel currents attract; antiparallel currents repel.



A current in a wire exerts no force on a **stationary** charge, only a moving charge.



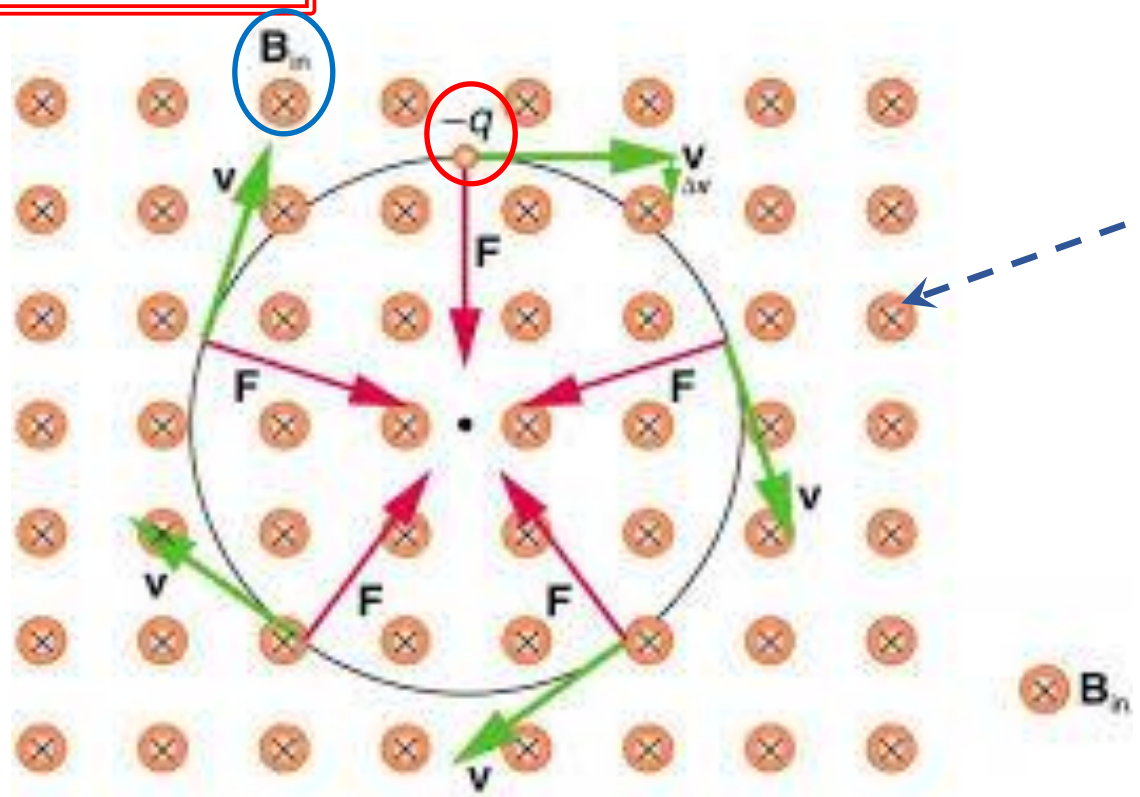
Magnetic Fields & Forces

$$\vec{F}_{\text{magnetic}} = Q\vec{v} \times \vec{B}$$ Lorentz force law (in vector notation)

$$\vec{F}_{\text{electric}} = Q\vec{E}$$

speed is magnitude of velocity.

$$|\vec{\text{velocity}}| = \text{speed}$$



The $\vec{B}$-field points into the plane.
The force ($\vec{F}$) is perpendicular to the velocity $\vec{v}$ and $\vec{B}$.
A constant magnetic field causes the particle to move in a circle

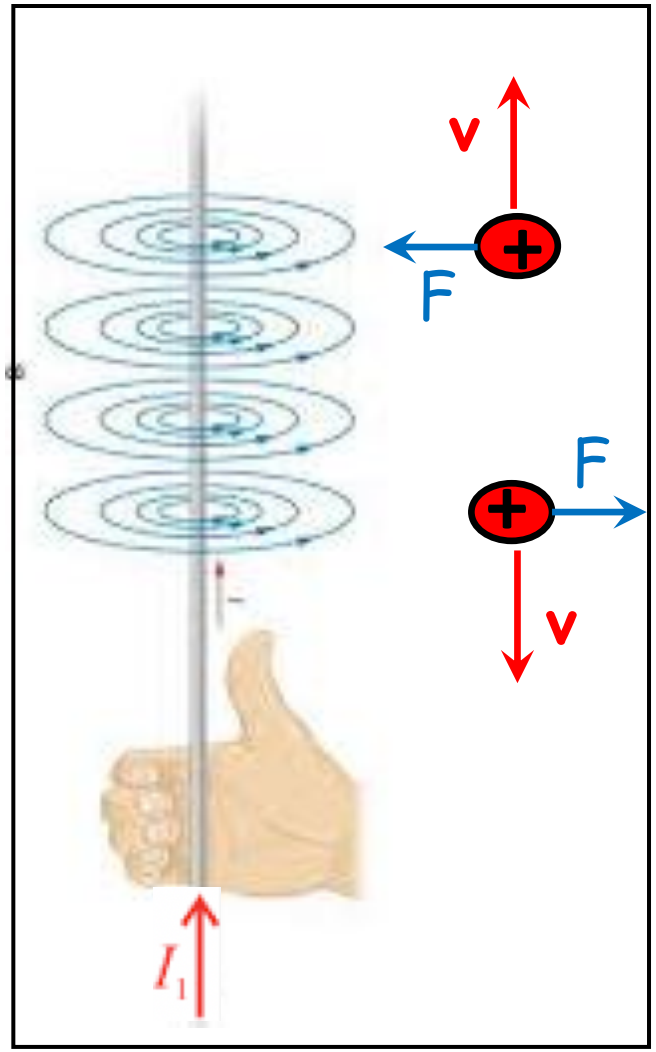
The speed is constant!
No change in kinetic energy.

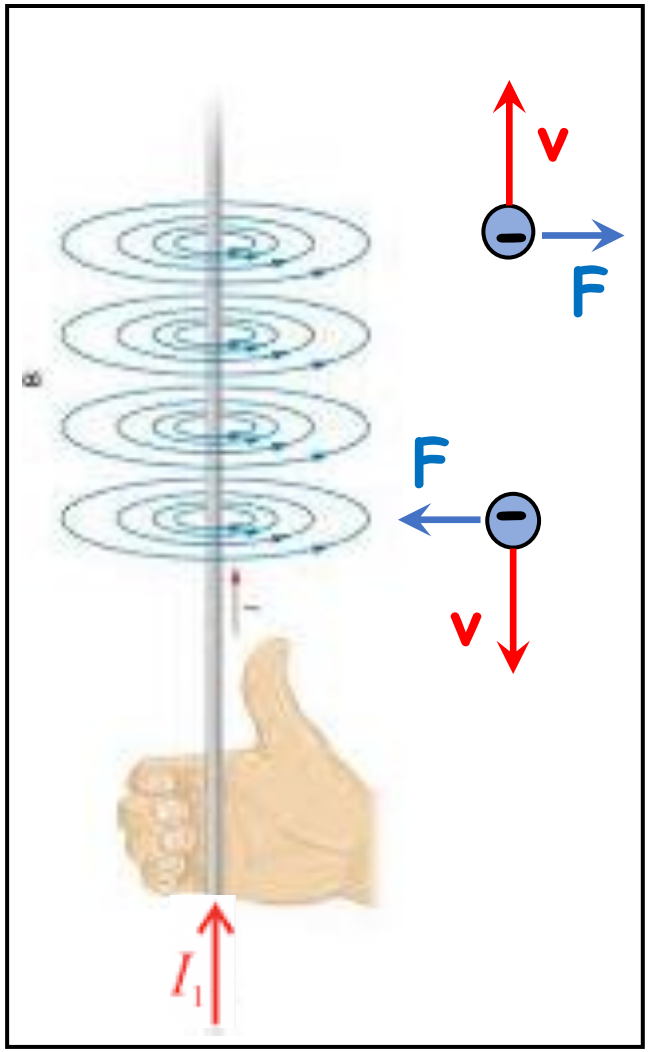
$$\text{KE} = \frac{1}{2}mv^2$$

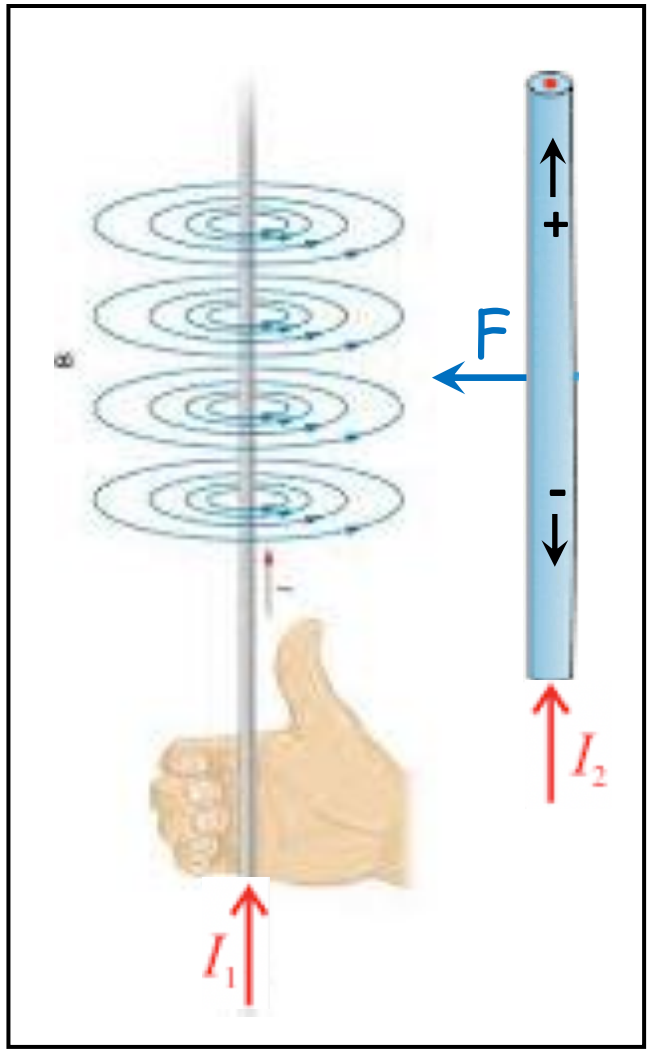
A magnetic field exerts a force on a moving charged particle, but does NO work on it.

Why parallel currents attract

The magnetic field of the wire exerts a force on a moving charged particle.

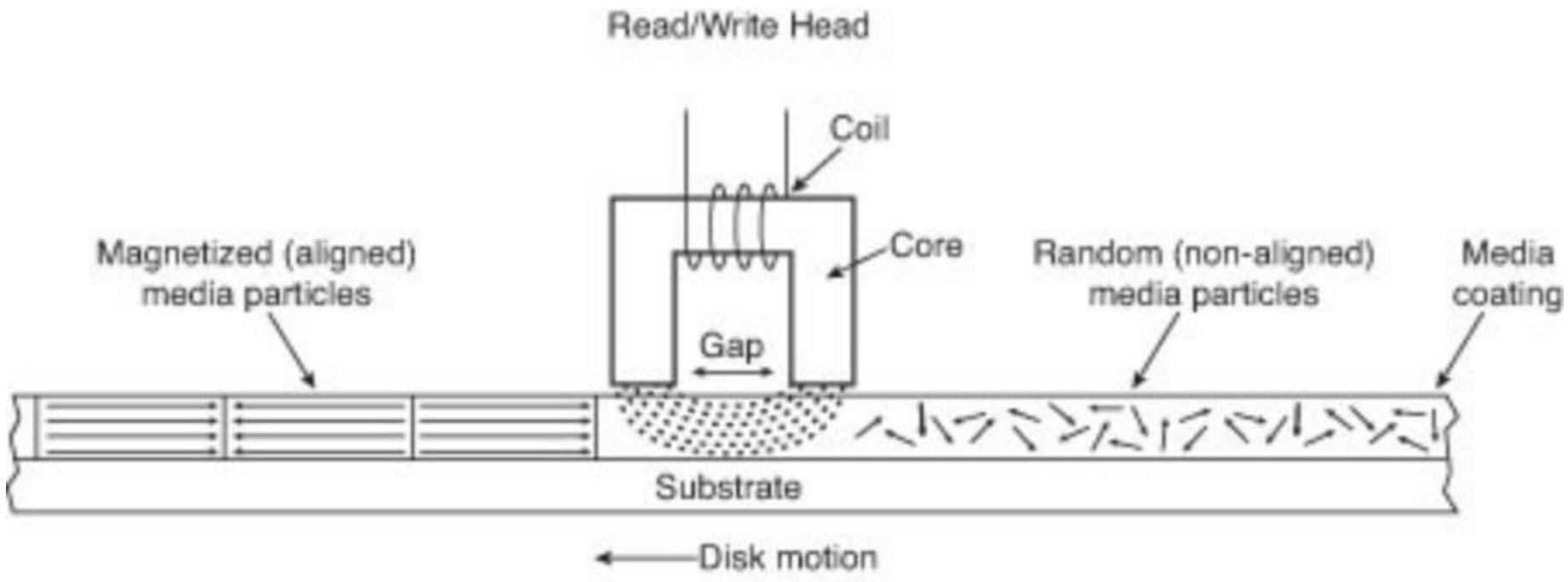




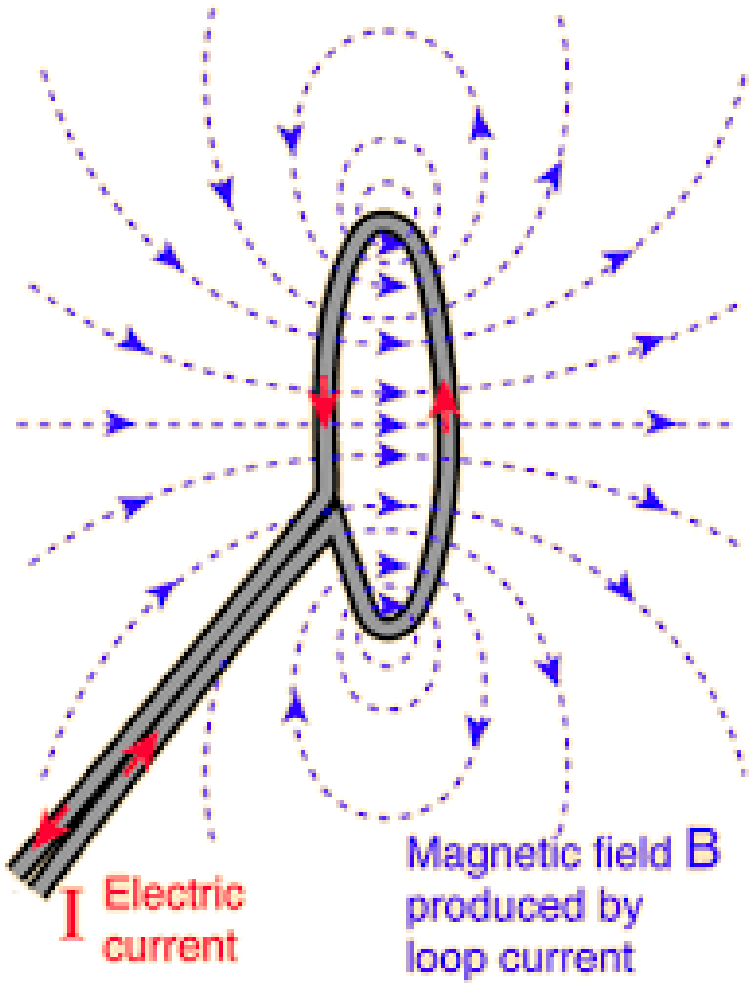


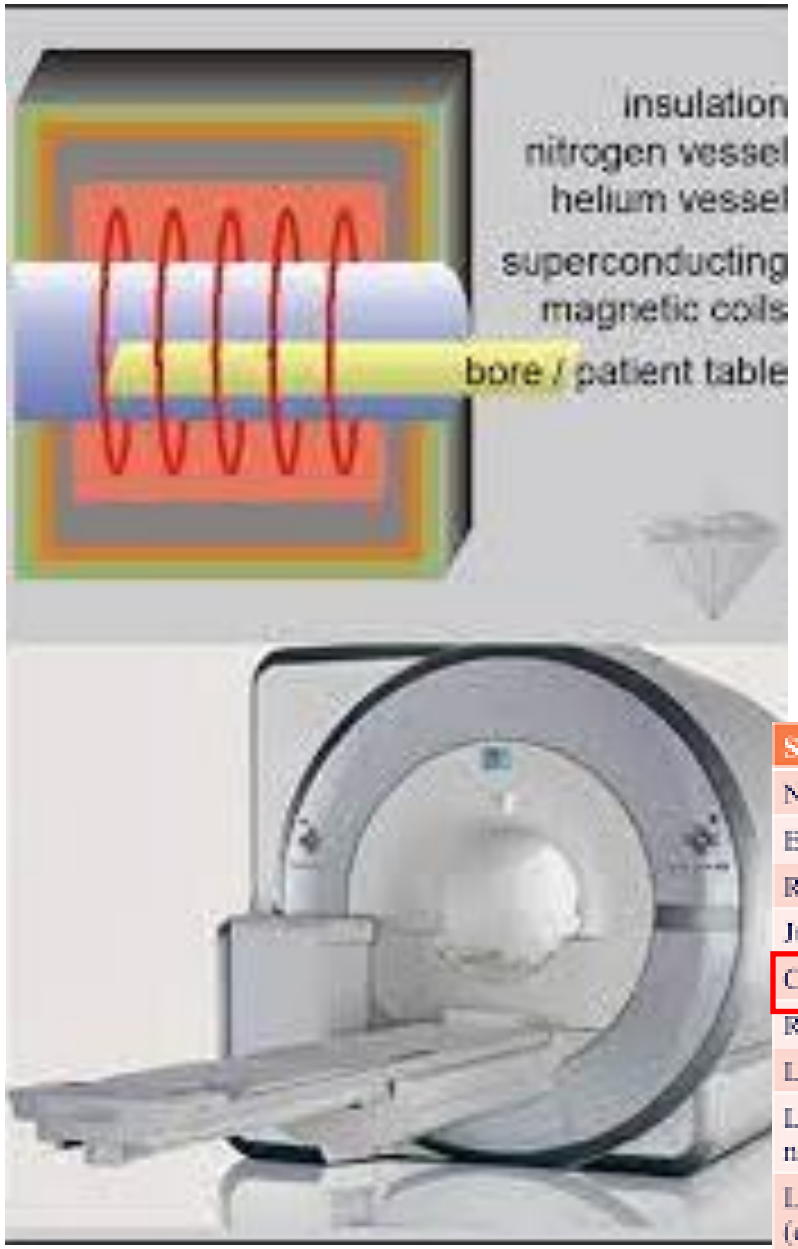
The current in the wire is positive charges ("holes") moving in the direction of the current I, or negative charges (electrons), moving in the opposite direction, or both.

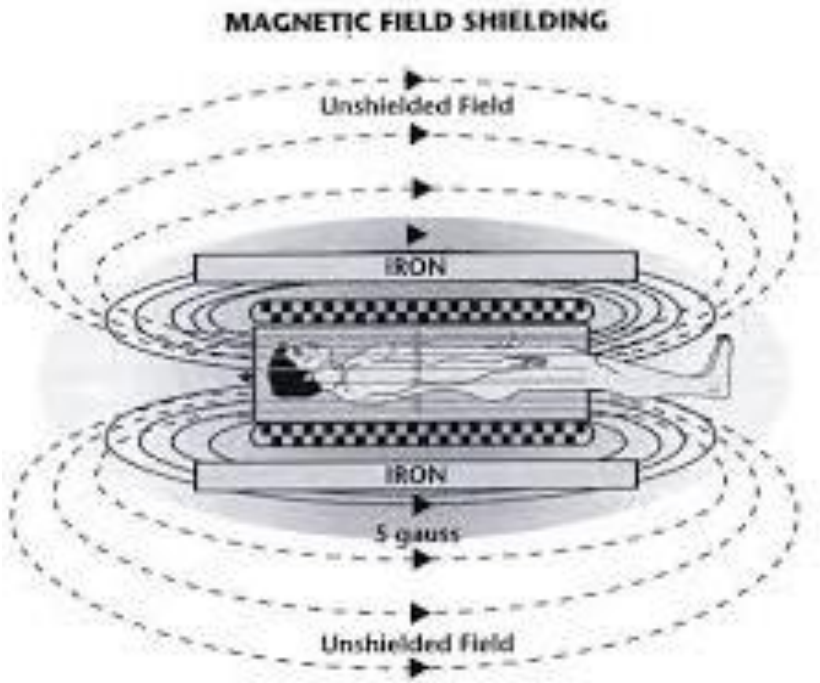
Amazing use of magnets – data storage



Information on our devices is stored as long sequences of 1's or 0's
e.g. 00101100101000111010001011110010 10…
If the magnetization points left, it is a 0; right is a 1
Each 0 or 1 (i.e. each "bit") is about 1 micron long.
One square cm is 100 million square microns.







Source	Approximate Magnetic Field Strength
Neuron depolarization (imaged by MEG)	0.5 pT (5 x 10^{-13} T)
Earth's magnetic field	0.5 G (50 μT)
Refrigerator magnet	50 G (5 mT)
Junkyard electromagnet	1 T
Clinical MRI scanners	0.5 - 3.0 T (typical)
Research MRI scanners (human)	7.0 T – 11.7 T
Laboratory NMR spectrometers	6 - 23 T
Largest pulsed field created in lab nondestructively	97 T
Largest pulsed field created in lab (destroying equipment but not the lab)	730 T

Even if annoying, the MRI B-field is not something we "feel" (unless we have steel rods in our body or pockets).

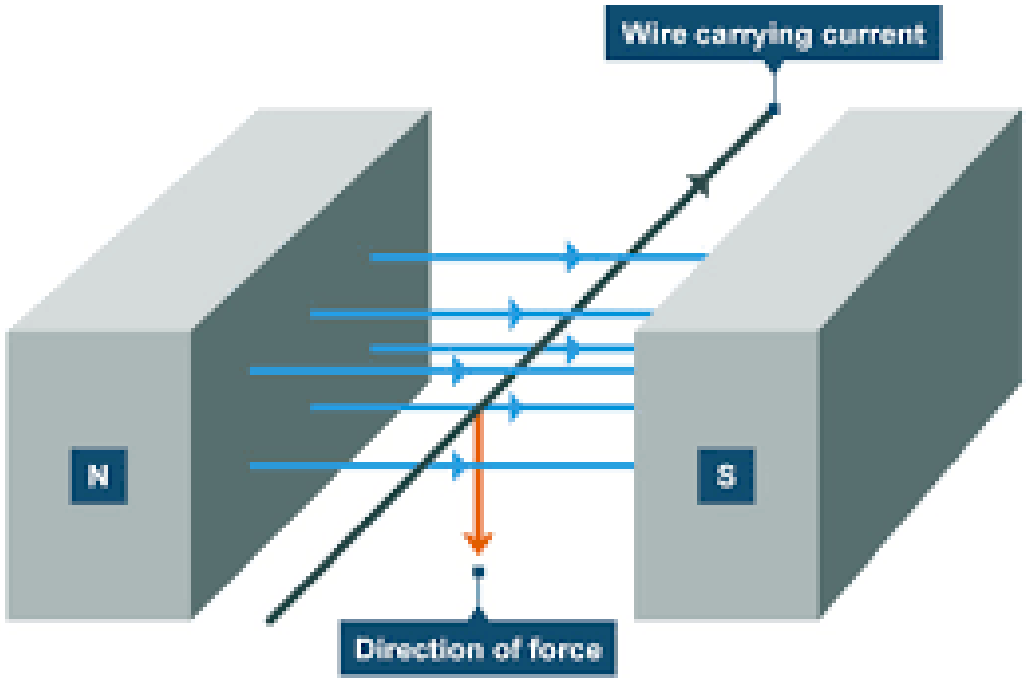
Magnetic clues draw police to ivory smuggling suspects



"Magnetic powder" is being applied with a "magnetic brush".

A magnetic field exerts a force on a current

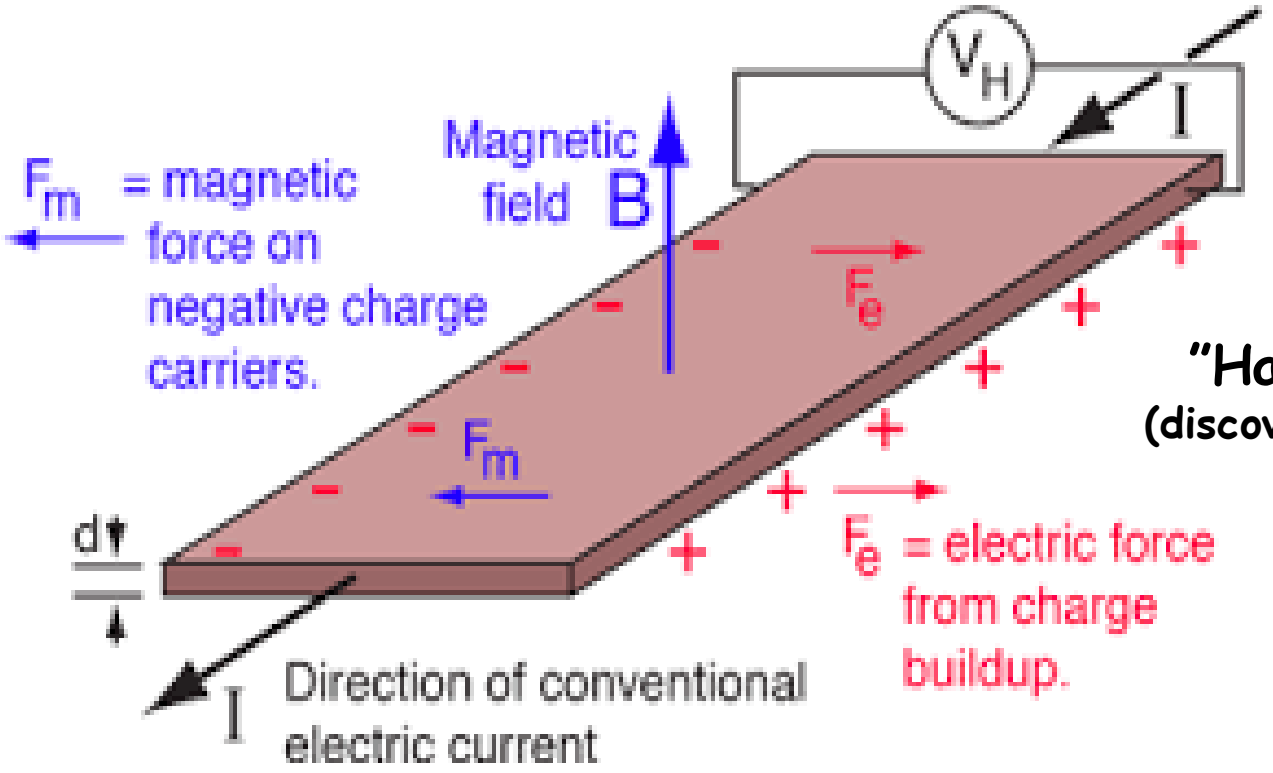
(if it is in a B-field perpendicular to the wire)



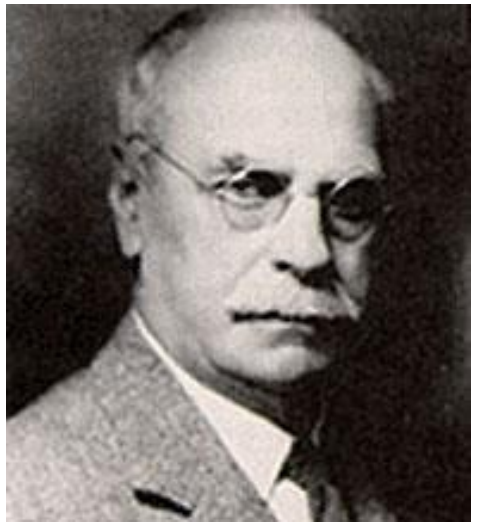
The wire gets accelerated, and acquires a velocity in the direction of the force

Therefore: **WORK is being done!**

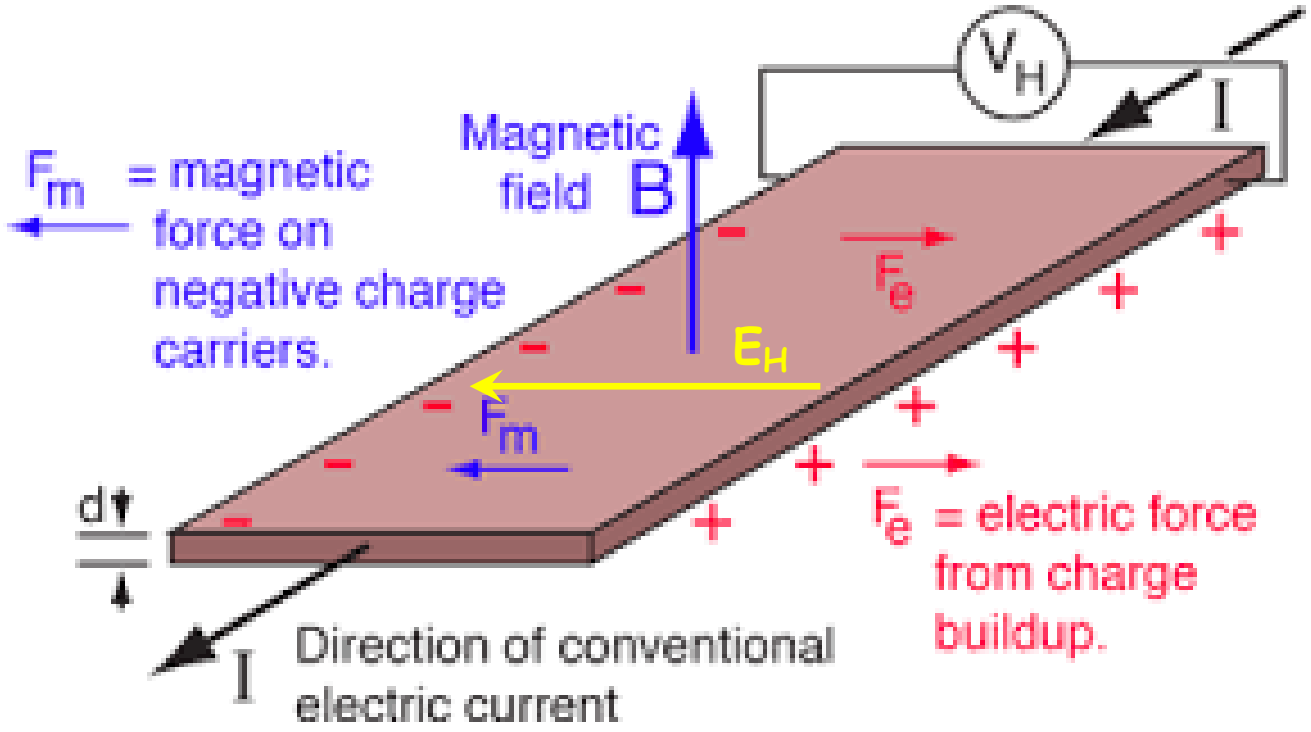
The "Hall voltage" V_H is measured by a voltmeter across the edges of a wire carrying current I, if there is a perpendicular magnetic field.



"Hall effect"
(discovered in 1879)



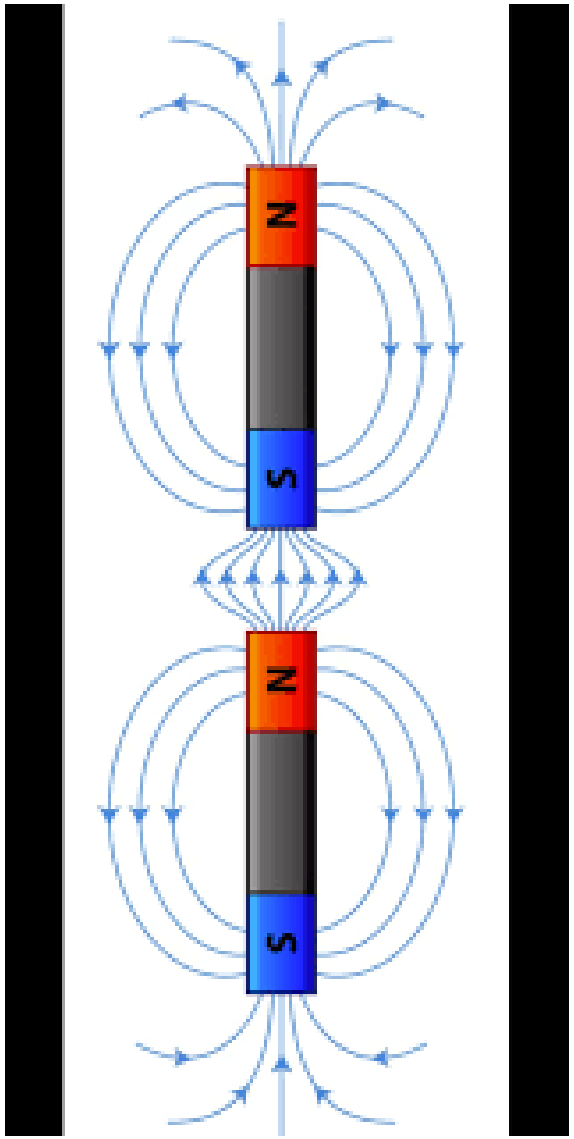
Edwin Hall (1855-1938)
born in Gorham, Maine



Electrons in the wire carry the current.
Charge accumulates on the edge.
and creates the "Hall field" E_H)
The electrons experience opposite transverse forces – magnetic and electric.
Current I in the wire flows un-deflected

Conclusion: Work is done on the wire, but the force on the wire is caused by the Hall field, not the B-field.

The magnetic field does no **direct** work on the wire.



opposite **poles** attract

A different example of work:
A magnet lifting another magnet.

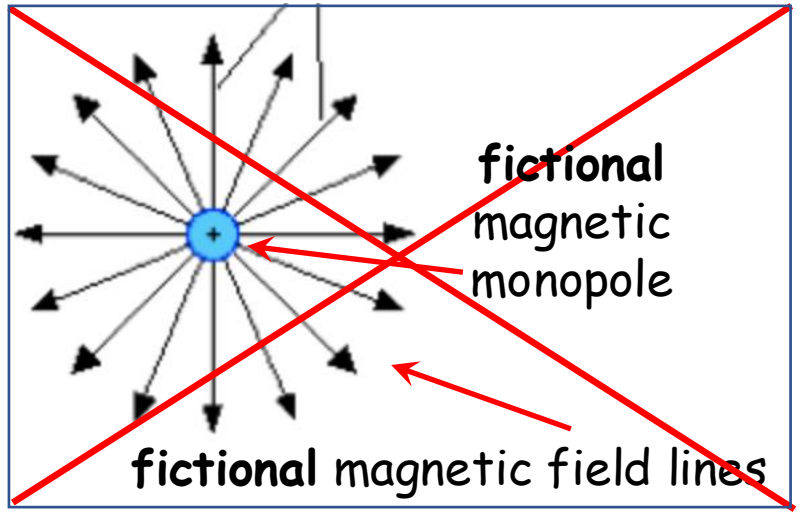
Hold the upper magnet in place, and
watch the lower magnet get pulled upward.

"Work" is being done.
But this "translational work" is not the only effect.
Since kinetic energy and gravitational potential energy are increasing, somewhere
some kind of energy
must be decreasing.
Why? A fundamental law:

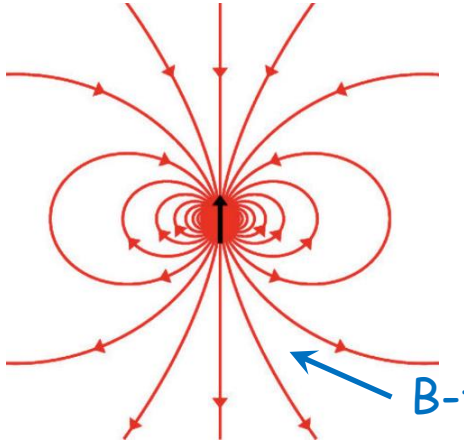
Total energy is conserved.

A (mysterious?) law of nature:

Magnetic monopoles do **NOT** exist.



but **Magnetic dipoles DO exist.**

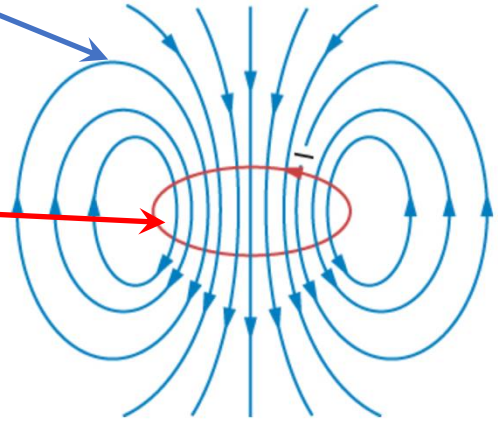


B-field of a particle with a magnetic moment

example:

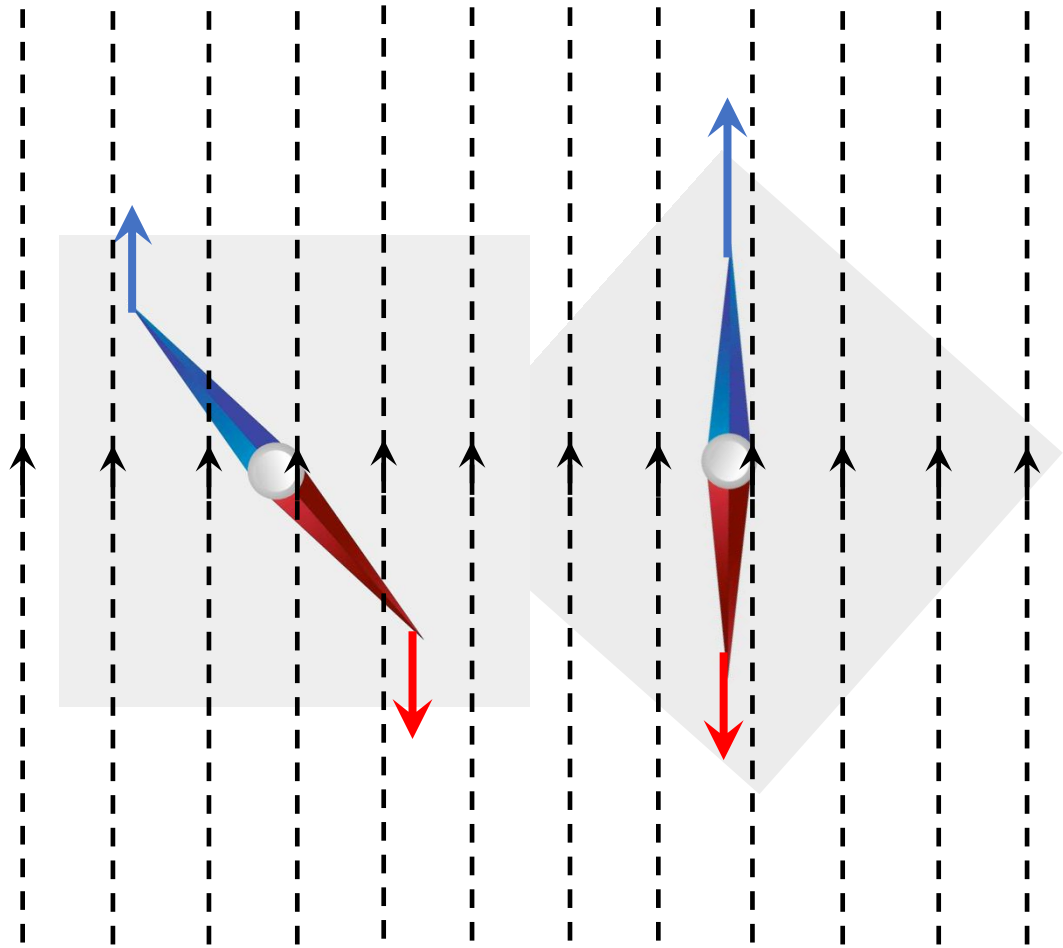
A neutron has no charge (Q=0)
but has a dipolar magnetic field.

The source is "***spin***" .



A **current loop**
creates a **dipolar** magnetic field
It is a **classical** model for a magnet,
and a model for an atom with an "***orbital***" magnetic moment.

Compass needles (magnets) in a uniform=homogeneous B-field



Total Force on the needles is 0.
No "translational" acceleration

Balanced forces re-orient the needle,
changing its kinetic energy,
doing "rotational" work.

$$\mathrm{KE} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

How can it be that the
magnetic field is NOT doing work?

The homogeneous field exerts a "torque" but no net force.

(more careful) **Definition of Work** - (alternative meanings)

If an object's ***kinetic + internal*** energy is changing, then some ***external*** agent is doing work on it.

Total energy $\mathcal{E} = T + U_{ext} + \mathcal{E}_{int}$ = constant

T = kinetic energy

U_{ext} = external potential energy

$\mathcal{E}_{int}$ = internal energy

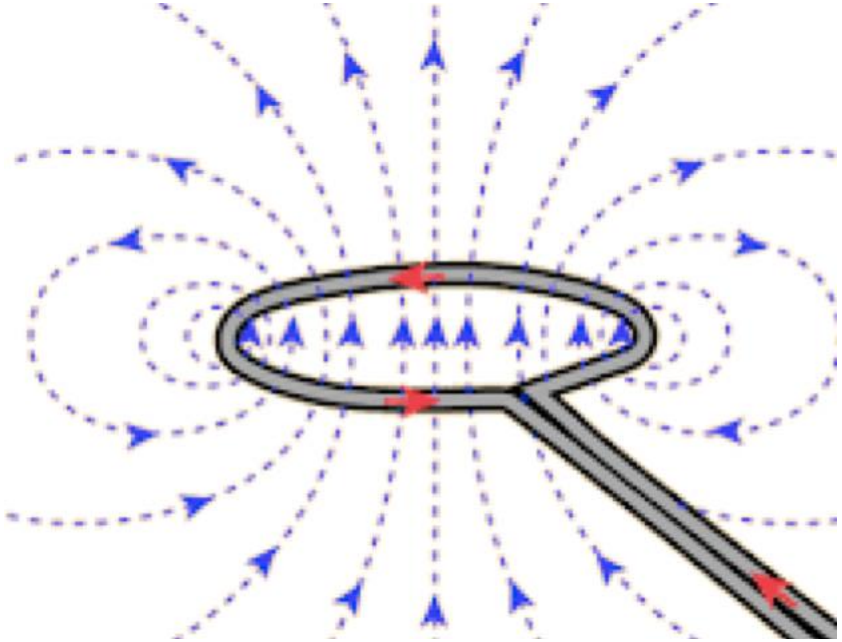
*A magnet's **translational** kinetic energy is unchanged by a constant magnetic field.*

*It's **rotational** kinetic energy* **does** change when the magnet realigns the magnet.

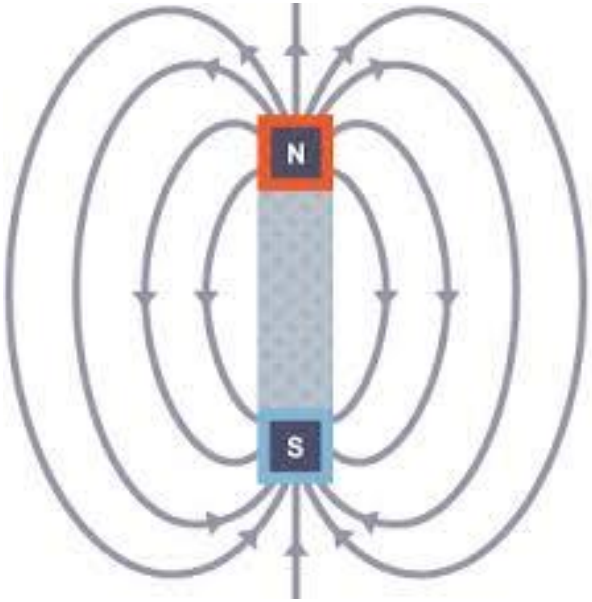
Question: **Does the magnet's "*hidden*" internal energy $\mathcal{E}_{int}$ also change?**

Answer: **For a *classical* model of a magnet, in a *constant* field B, $T + \mathcal{E}_{int}$ is constant as torque reorients the magnet. No Work!**

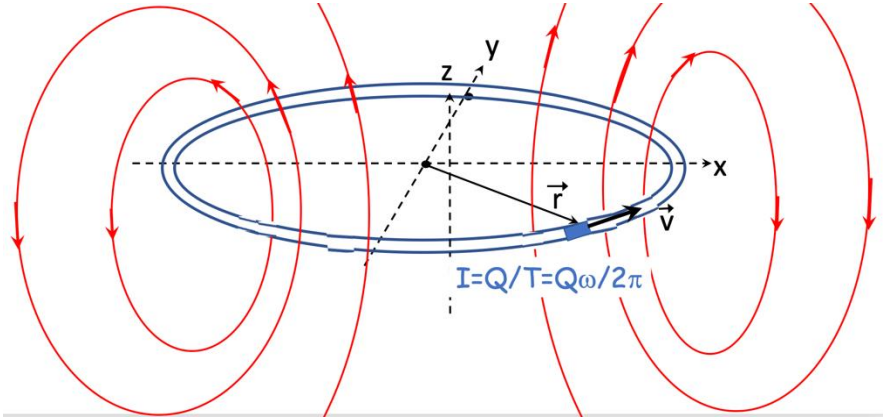
Models of a magnet



current-carrying wire



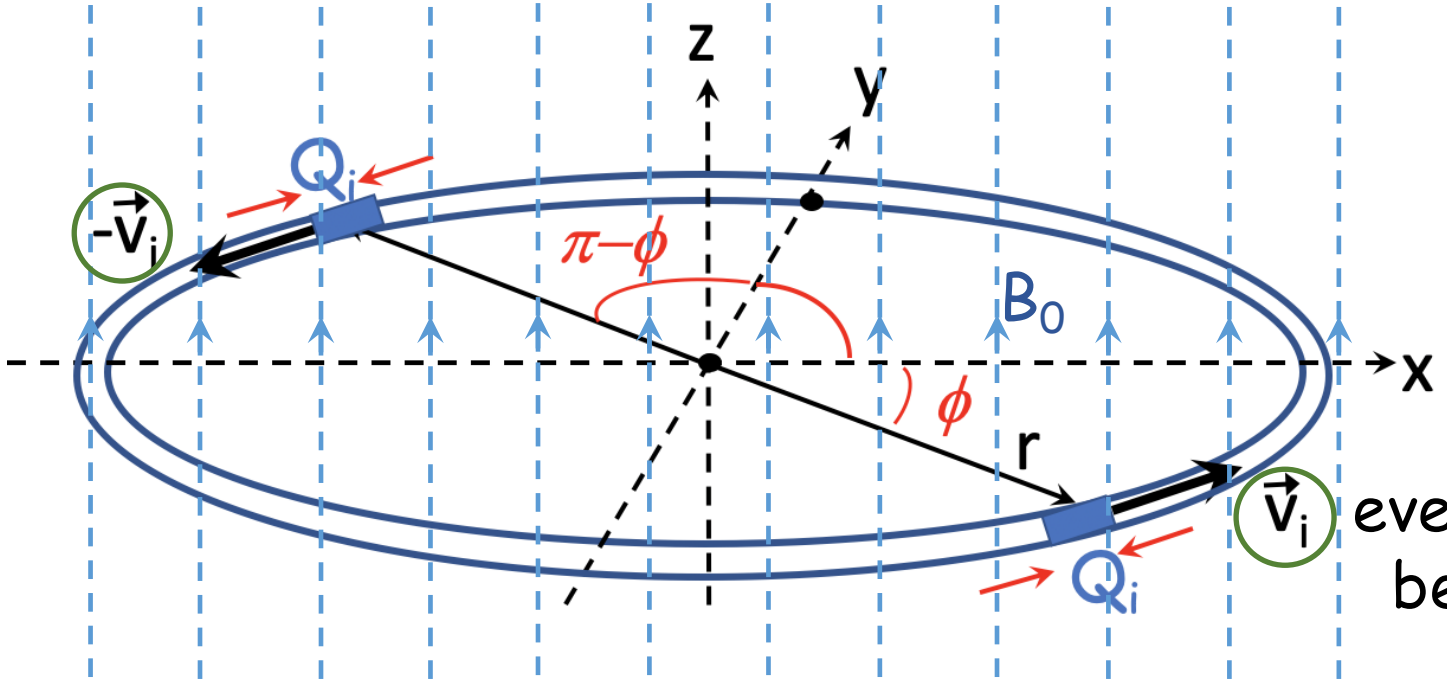
bar magnet



rotating charged loop

The rotating charged loop is a convenient classical model

model magnet: rotating charged ring



homogeneous Field B_0.

Lorentz force law:

$$\vec{F} = \left[\sum_i Q_i \vec{v}_i\right] \times \vec{B}_0$$

The sum [ ] = 0

even though the velocities are not zero.

because opposite velocities v_i cancel

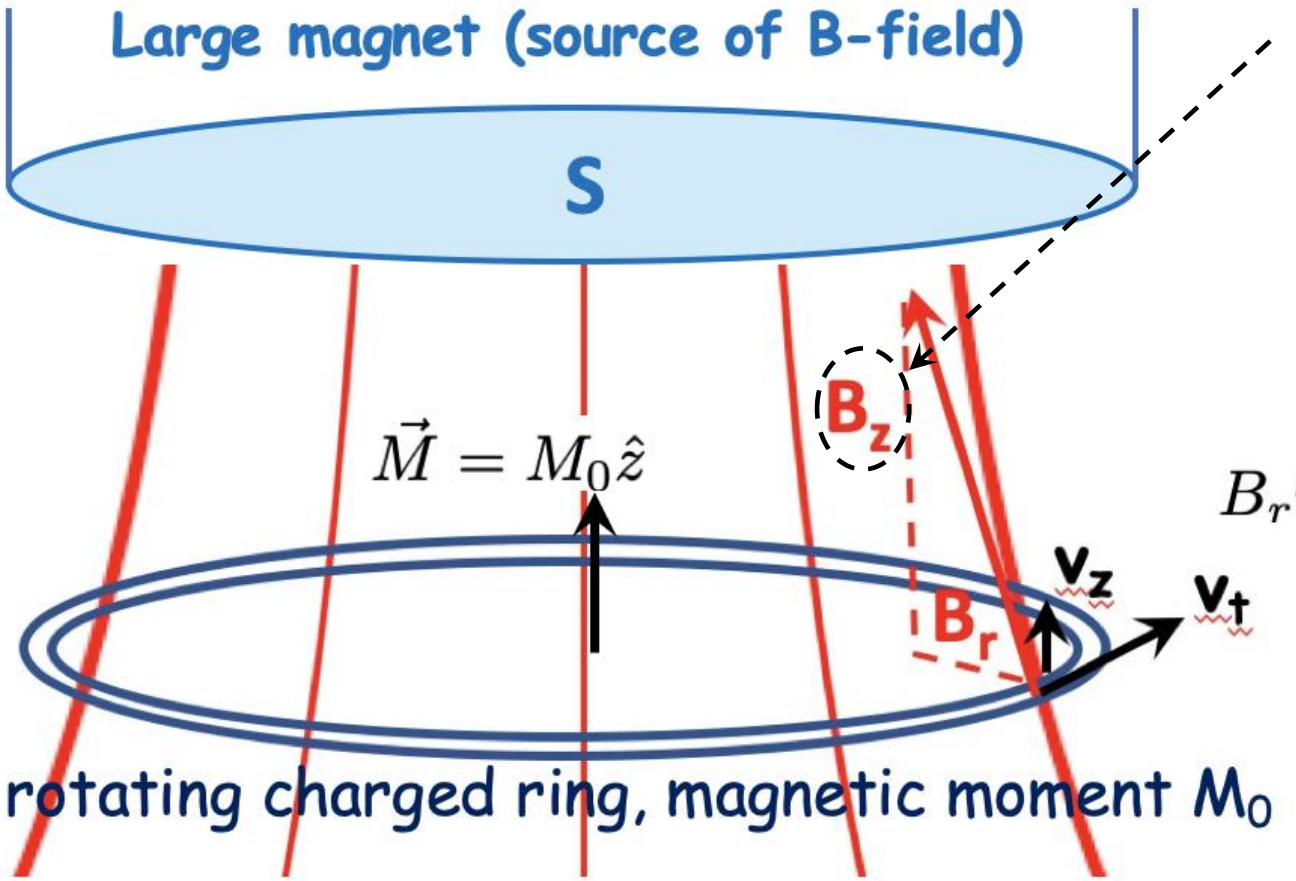
A homogeneous field $\vec{B}_0$ gives $\vec{F} = 0$ (**no force**) on a rotating charged ring magnet.

This is a classical proof a correct **general statement**:

Homogeneous B-fields do no *translational* work on magnets.

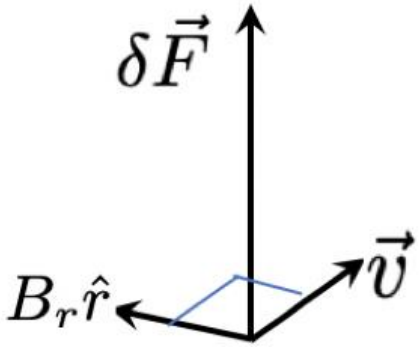
(Already noticed in the case of the compass needle.)

Inhomogeneous Field

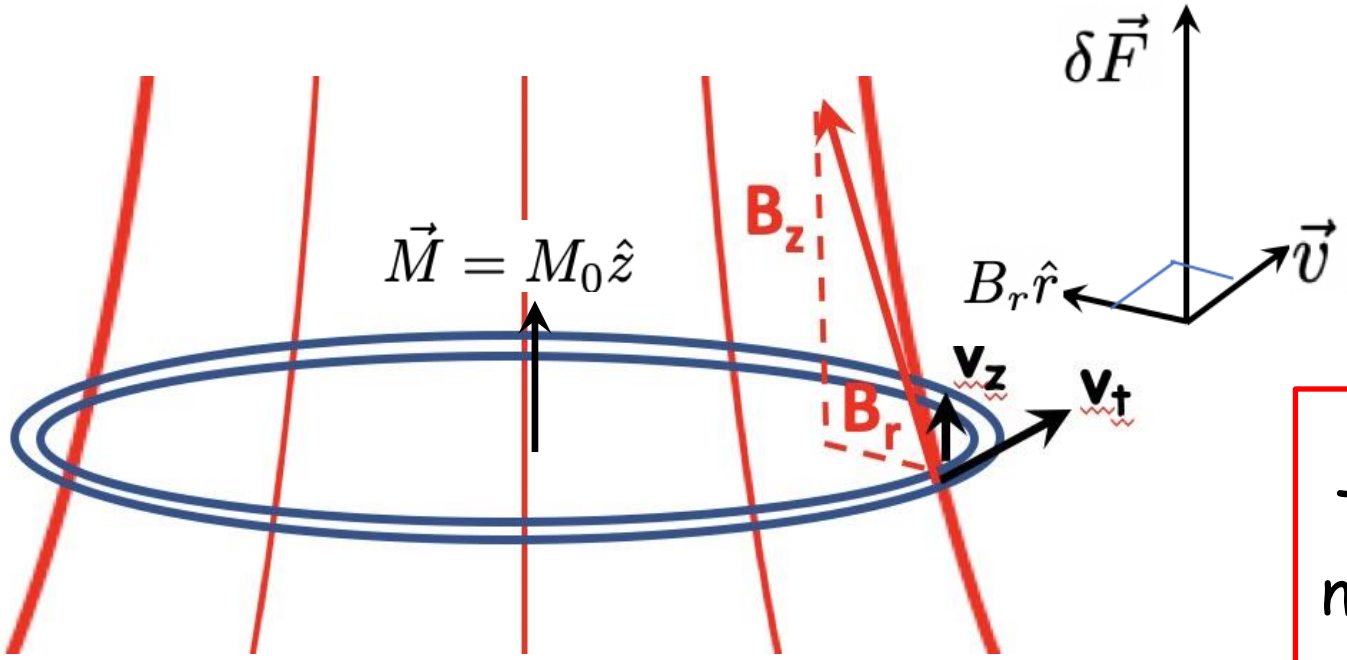


B_z gives a radially inward force on each piece.

Total B_z-force sums to zero.



A small magnet is attracted upward by the B-field of a large magnet.
The **radially inward field B_r** causes the upward force.
If the ring has also an upward velocity v_z, then B_r **also** causes a smaller force that slows down the ring's rotational speed v_t.



For us nerds: the force on a magnet is $\vec{F} = \vec{\nabla}(\vec{m}\cdot\vec{B})$ where our magnetic moment is $\vec{m} = M_0\hat{z}$, so the Force here is

$$\vec{F}_{\text{nerd}} = M_0\frac{dB_z}{dz}\hat{z}$$

The ring accelerates upward. The **radial forces** cancel and give no net (*translational*) acceleration. But they do alter rotational speed.

Lorentz forces on a piece of ring (charge Q_i):

$$\vec{F}_{z,i} = Q_i\vec{v}_{t,i}\times\vec{B}_r = Q_i v_{t,i}B_r\hat{z} \quad \text{(upward)}$$

$$\vec{F}_{r,i} = Q_i\vec{v}_{z,i}\times\vec{B}_r = -Q_i v_z B_r\hat{v}_{t,i} \quad \text{(opposite to } \vec{v}_{t,i}\text{)}$$

Sum the forces on all pieces i:

$$\vec{F}_{z,\text{tot}} = Qv_tB_r\hat{z};\quad \vec{F}_{r,\text{tot}} = 0$$

[This agrees with F_{nerd} after some effort, using $\vec{\nabla}\cdot\vec{B} = 0$]

"Torque" is the concept needed to deal with changes of rotational motion; "force" won't do it.

Generalize Newton's law to deal with rotation:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$\vec{\mathcal{T}} = \overleftrightarrow{\mathcal{I}} \cdot \vec{\alpha} = \overleftrightarrow{\mathcal{I}} \cdot \frac{d\vec{\omega}}{dt}$$

$\vec{\mathcal{T}}$ = torque vector

$\overleftrightarrow{\mathcal{I}}$ = moment of inertia tensor

$\vec{\omega}$ = angular velocity vector

$\vec{\alpha} = \dfrac{d\vec{\omega}}{dt}$ = angular acceleration vector

The moment of inertia tensor of the ring is

$$\begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} = \begin{pmatrix} mr_0^2/2 & 0 & 0 \\ 0 & mr_0^2/2 & 0 \\ 0 & 0 & mr_0^2 \end{pmatrix}$$

For the ring $\omega_z=\omega=v_t/r_0$

$$\vec{\mathcal{T}} = \sum_i \vec{r}_i \times \vec{F}_i = -Qr_0 v_z B_r \hat{z}$$

$$\mathcal{T}_z = \mathcal{I}_{zz}\frac{d\omega}{dt}$$

Rate of doing work – a cleaner concept than work.

Translational version

$$\frac{dW_{\text{trans}}}{dt} = \vec{F} \cdot \vec{v}$$ = force times rate of change of position in the direction of the force

Since $\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$, we have $\frac{dW_{\text{trans}}}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{d\text{KE}_{\text{trans}}}{dt}$

Angular version

$$\frac{dW_{\text{rot}}}{dt} = \vec{\mathcal{T}} \cdot \frac{d\vec{\omega}}{dt}$$ = torque times rate of change of angle in the direction of the torque

Since $\vec{\mathcal{T}} = \overleftrightarrow{\mathcal{I}} \cdot \vec{\omega}$, we have $\frac{dW_{\text{rot}}}{dt} = \frac{d}{dt}\left(\frac{1}{2}\vec{\omega} \cdot \overleftrightarrow{\mathcal{I}} \cdot \vec{\omega}\right) = \frac{d\text{KE}_{\text{rot}}}{dt}$

Combined version

$$W = W_{\text{trans}} + W_{\text{rot}} \qquad \text{KE} = \frac{1}{2}mv^2 + \frac{1}{2}\vec{\omega} \cdot \overleftrightarrow{\mathcal{I}} \cdot \vec{\omega} \qquad \frac{dW}{dt} = \frac{d\text{KE}}{dt}$$

$$\frac{dW(\text{on object})}{dt} = \frac{d\text{KE}(\text{of object})}{dt}$$

The total force on the ring is $\vec{F} = Qv_t B_r \hat{z}$

The total torque on the ring is $\vec{T} = -Qr_0 v_z B_r \hat{z}$

The total rate of doing work is $\frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}\vec{\omega}\cdot\overleftrightarrow{\mathcal{I}}\cdot\vec{\omega}\right) = \vec{v}\cdot\vec{F} + \omega\cdot\vec{T}$

$$\frac{dW}{dt} = v_z(Qv_t B_r) - \omega(Qr_0 v_z B_r) = 0 \text{ since } v_t = \omega r_0$$

The rotating ring is accelerated by the B-field of the big magnet,
NO WORK IS DONE. Negative rotational work cancels positive translational work.
Total kinetic energy of the ring stays constant.

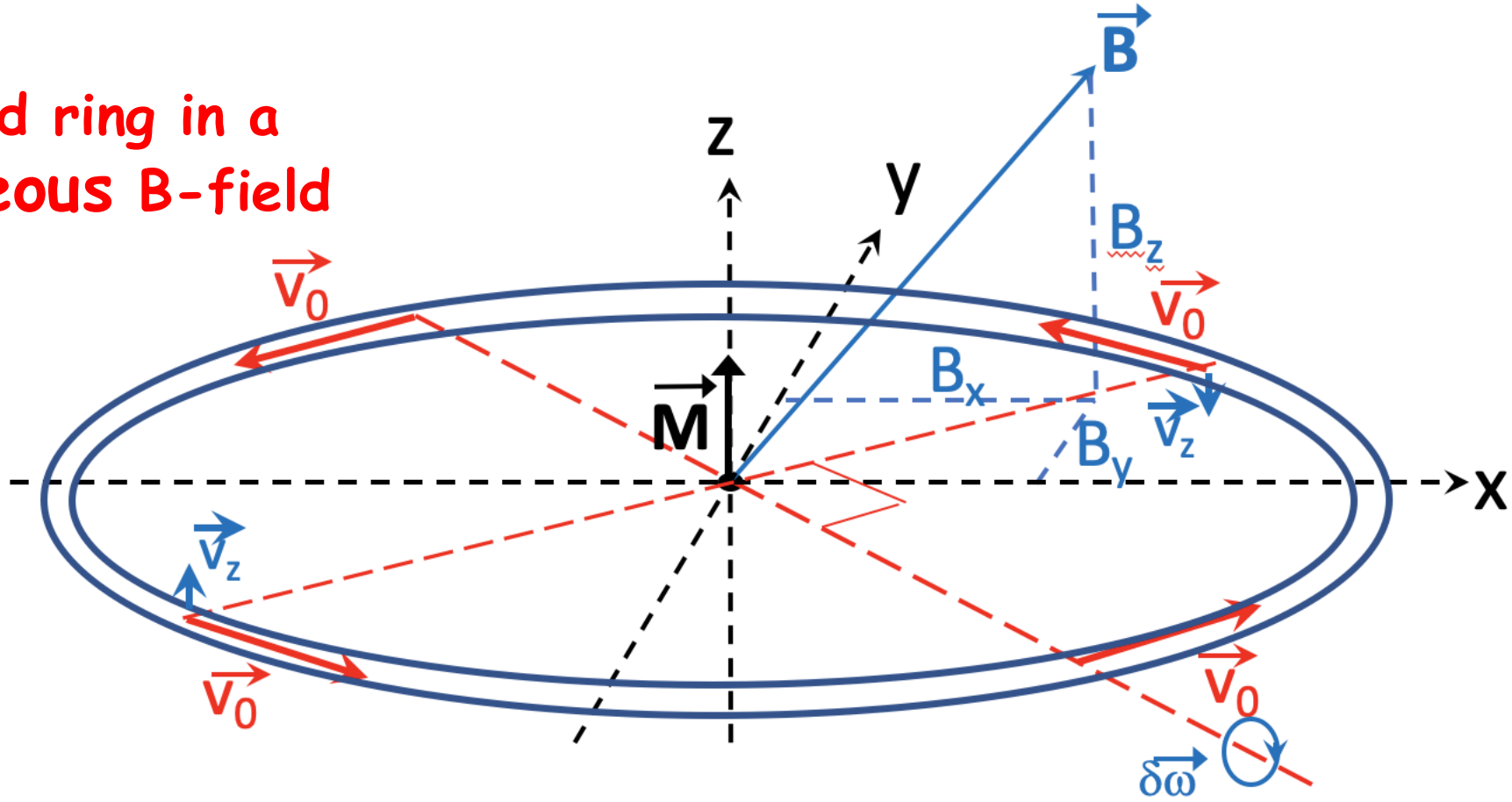
If it had been a real magnet instead of the ring, you wouldn't **see**
rotational kinetic energy.
It is hidden inside in the magnet.
Then it is not considered rotational ***kinetic*** energy, but ***"internal"*** energy.
A **"fuller version"** definition of work is appropriate.
Work **on** an object = change of total kinetic energy plus internal energy **of** the object.

A different kind of magnetic work

A magnetic field $\vec{B}$ exerts a torque $\vec{T}$ on a magnet with magnetic moment $\vec{M}$.

$$\vec{T} = \vec{M} \times \vec{B}$$

Misoriented ring in a homogeneous B-field

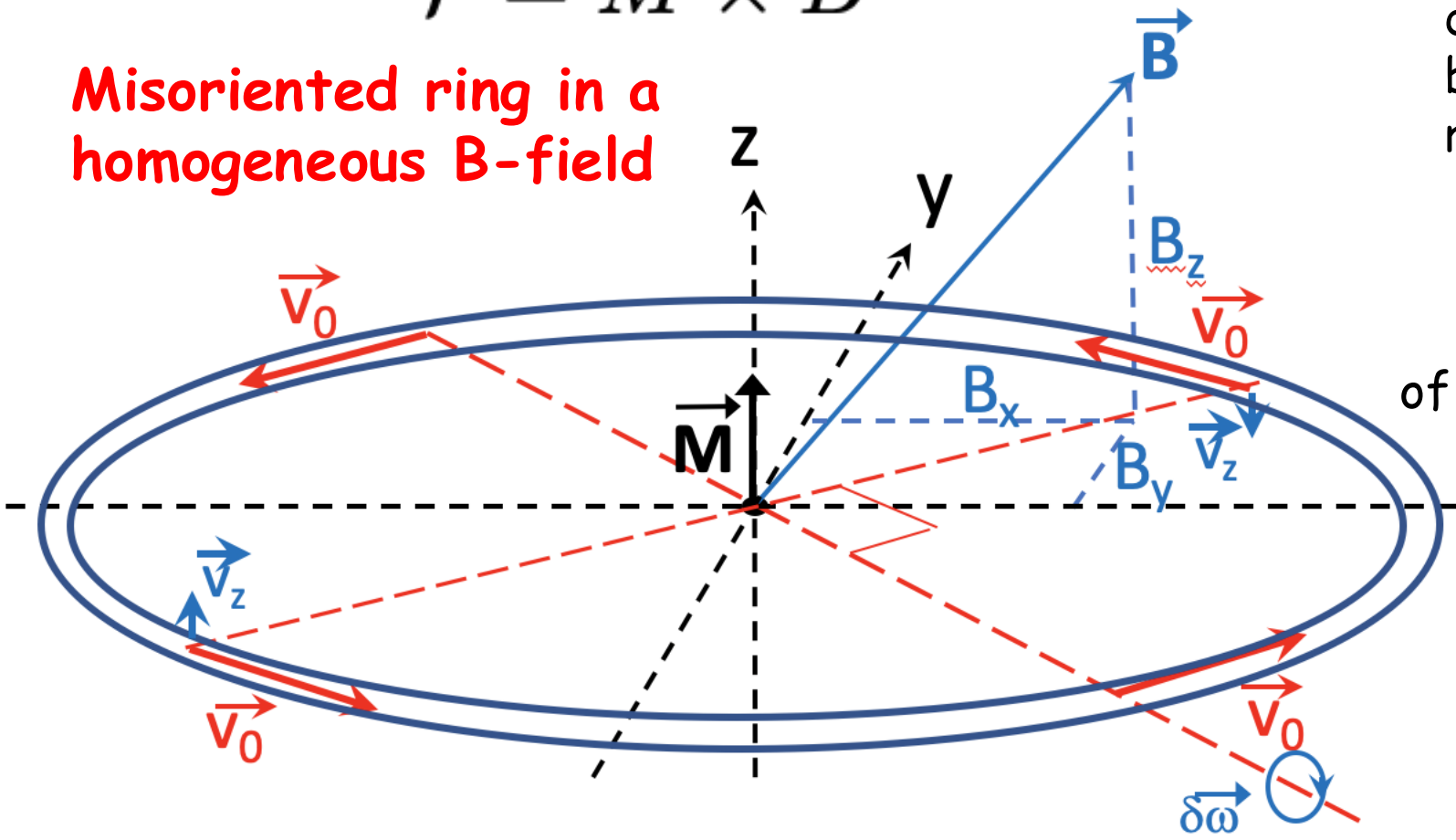


A different kind of magnetic work

A magnetic field $\vec{B}$ exerts a torque $\vec{T}$ on a magnet with magnetic moment $\vec{M}$.

$$\vec{T} = \vec{M} \times \vec{B}$$

Misoriented ring in a homogeneous B-field



There is no net force, because the field is homogenous. But the field **tries to orient** the ring so that

$$\vec{M} \parallel \vec{B}$$

This causes pieces of the ring to develop small velocities v_z (shown in blue), in addition to the usual ring velocities v_0 (shown in red).

The ring therefore develops a rotation rate $\vec{\omega} = \omega_0 \hat{z} + \delta\vec{\omega}$, where $\delta\omega = v_z/r_0$ rotates the plane of the ring. It is small compared to the rate $\omega_0 = v_0/r_0$ around the z axis.

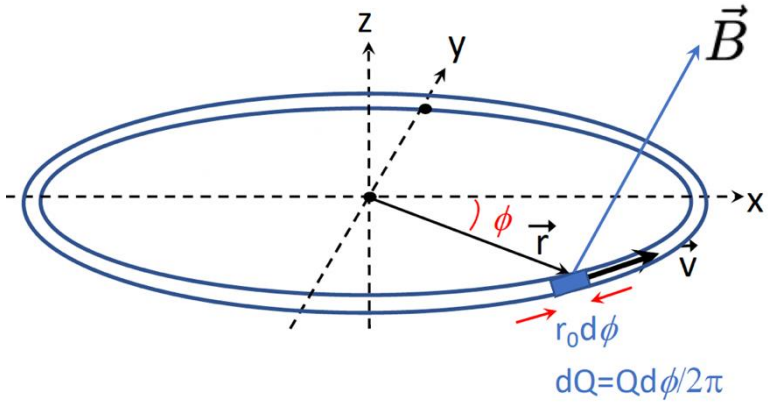
Newton's Law

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$$\vec{T} = \overleftrightarrow{\mathcal{I}} \cdot \frac{d\vec{\omega}}{dt}$$

Rate of doing work: $\frac{dW}{dt} = \vec{v} \cdot \vec{F} + \vec{\omega} \cdot \vec{\mathcal{T}}$

The homogeneous field give no force, but its *torque* does *rotational* work.



Quick summary of calculated rotational work

$$\vec{\mathcal{T}} = \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times (Q_i \vec{v}_i \times \vec{B}_0)$$

$\vec{v}_i = \vec{\omega} \times \vec{r}_i$ and $\vec{\omega} = (\omega_x, \omega_y, \omega_z)$

$\omega_z \approx \omega_0$ and ω_x, ω_y are small

$$\frac{dW_{\mathrm{rot},a}}{dt} = \omega_x \mathcal{T}_x + \omega_y \mathcal{T}_y = -\frac{Qr_0^2}{2}\omega_z(\omega_x B_{0y} - \omega_y B_{0x})$$

$$\frac{dW_{\mathrm{rot},b}}{dt} = \omega_z \mathcal{T}_z = +\frac{Qr_0^2}{2}\omega_z(\omega_x B_{0y} - \omega_y B_{0x}).$$

$$\mathcal{T}_x = -\frac{Qr_0^2}{2}\omega_z B_{0y}; \quad \mathcal{T}_y = +\frac{Qr_0^2}{2}\omega_z B_{0x};$$

These are large and reorient axis of ring toward B.

$$\mathcal{T}_z = \frac{Qr_0^2}{2}(\omega_x B_{0y} - \omega_y B_{0x})$$

This is small and alters the rotation frequency ω_z.

Work done by the large torques, reorienting the axis of the ring.

$$\frac{dW_{\mathrm{rot},a}}{dt} = \omega_x \mathcal{T}_x + \omega_y \mathcal{T}_y = -\frac{Qr_0^2}{2}\omega_z(\omega_x B_{0y} - \omega_y B_{0x})$$

Work done by the small torque, altering the ring's rotational frequency ω_z.

$$\frac{dW_{\mathrm{rot},b}}{dt} = \omega_z \mathcal{T}_z = +\frac{Qr_0^2}{2}\omega_z(\omega_x B_{0y} - \omega_y B_{0x}).$$

These cancel each other! No rotational work is done!

Conclusion: magnetic fields do no work on the rotating ring model of a magnet.

Real Magnets?

My personal opinions (for now, anyway):

1. Neglect ***spin*** magnetic moments at first. **Classical** theory shows:
    a. The magnetic field *apparently* does work:
       translational [oriented parallel to an inhomogeneous field]
       rotational [oriented away from parallel to a homogeneous field]
    b. No **net** work is actually done. But, the compensating change of rotational frequencies is ***hidden*** inside the magnet. And the inside behavior is complex! The work inside *might* be reassignable to some electrical effect.
2. Continue to neglect spin. Do **quantum** theory.
    a. I did this in 1964 (for one case) and got the same result.
    b. It should be fun to repeat this now.
3. Do a similar analysis for quantum ***spins***. Should be easy. I will try soon!
4. Who cares whether we assign the work to the magnetic field?
   Magnets are good at doing work. The issue is the word "WORK".

Thank you

And thanks especially to

Prof. Robert Romer
Prof. Haven Whiteside