Homework set #9, due April 22, 11:59 pm

1) Using a classical model of the H-atom, calculate the energy that is radiated in one period in units of the binding energy of the electron. Use the Larmor formula.

2) Consider an antenna of length d with current $I(z,t) = I_0\left(1 - \frac{2|z|}{d}\right)e^{-i\omega t}$

To leading non vanishing order in kd calculate

a) The angular distribution of the radiated power.

b) The total radiated power.

3) An antenna of negligible thickness and length d carries a sinosoidal current with wave length d.

a) Calculate the radiated power per unit of solid angle a a function of the angle. Make a plot.

b) Calculate the total radiated power

c) Calculate the angular distribution of the radiation in the dipole approximation and compare with the exact result of a)

4) Consider a rotating electric dipole of two charges connected by a massless rod of length d. It rotates counterclockwise in the xy plane with angular frequency $\omega = ck$. At $t=0$ the electric dipole moment is $P_0 = qd\hat{x}$.

a) Calculate $A(\vec{r},t)$ in the radiation zone and show that it can be written as

$$A(\vec{r},t) = \frac{P_0\omega}{r}(\hat{\phi} - i\hat{\rho})\, e^{i(\phi - \omega t + kr)}$$

(r,θ,φ) is $\vec{r}$ in spherical coordinates

$$\begin{pmatrix}\hat{\rho}\\ \hat{\phi}\end{pmatrix} = \begin{pmatrix}\cos\varphi & \sin\varphi\\ -\sin\varphi & \cos\varphi\end{pmatrix}\begin{pmatrix}\hat{x}\\ \hat{y}\end{pmatrix} \qquad (\text{also } \varphi = \phi)$$

b) Calculate $\vec{B}$ and $\vec{E}$ in the radiation zone

c) Calculate the radiated power per unit solid angle as a function of θ.
(Use the complex expression for the Poynting vector that already has been time averaged