Homework set #8

due April 15, 11.59pm (2021)

1. Calculat the energy flux perpendicular to the surface of the incoming wave (at angle θ to the normal, the reflected wave and the transmitted wave. We have that $\varepsilon = \mu = 1$ for $x < 0$ and $\varepsilon > 1$, $\mu = 1$ for $x > 0$. Take the polarization vector in the scattering plane.

2. We consider light that moves throug a mediam with dielectric constant $\varepsilon_{ij} = \varepsilon_0 \delta_{ij} + i\varepsilon_1 \varepsilon_{ij3}$
   a) show that left and right polarized light can propagate in the 3-direction without absorbtion. Are there conditions on ε_0 and ε_1 for this to happen?
   b) Calculate the rotation of a plane polarized beam in the 3-direction.

3. Consider an incoming wave with $\vec{e}$ and $\vec{k}$ from the vacuum into a region with $\varepsilon > 1$ for $z < 0$. The reflected wave has $\vec{e}_R$, $\vec{k}_R$ and the transmitted wave $\vec{e}_T$, $\vec{k}'$
   a) Show that $\vec{k}$, $\vec{k}'$ and $\vec{e}_R$ are coplanar
   b) show that the incident angle and the reflected angle are equal
   c) show that $\vec{e}_R$ and $\vec{e}_T$ are in the plane of $\vec{k}$ and $\vec{e}$.

4. The dielectric constant satisfies $\varepsilon(\omega)$ is analytic for $\operatorname{Im}\omega > 0$ and $\varepsilon(\omega) \to 1$ for $\omega \to \infty$ and $\operatorname{Im}\varepsilon(\omega) > 0$ for $\omega > 0$. For $\omega \in \mathbb{R}$, we have $\varepsilon^*(\omega) = \varepsilon(-\omega)$. Using among others Cauchy's theorem show that
   a) $\operatorname{Im}\varepsilon(\omega) < 0$ for $\omega < 0$ on the real axis
   b) $\varepsilon(\omega)$ has no zeros in the upper half ω plane, so $\sqrt{\varepsilon(\omega)}$ is analytic there
   c) Define $\sqrt{\varepsilon(\omega)}$ as having a positive imaginary part for $\omega \in \mathbb{R}^+$. Show $\operatorname{Im}\sqrt{\varepsilon(\omega)} < 0$ for $\omega \in \mathbb{R}^-$
   d) show that $\frac{\operatorname{Im}\sqrt{\varepsilon}}{\operatorname{Re}\sqrt{\varepsilon}} > 0$ for $\omega \in \mathbb{R}^+$ so that $\sqrt{\varepsilon} \to 1$ for $\omega \to \infty$.