

Homework #7, due 4-8-2021, 11.59 pm

1. An infinitely long wire along \hat{z} with charge density λ is surrounded by a cylindrical shell of radius a and charge density $\sigma = \frac{\lambda}{2\pi a}$

The cylinder can rotate freely about the z -axis but is initially at rest in a constant external field $\vec{B}_{ext} = B_0 \hat{z}$

- a) at $t=0$, we slowly reduce the magnetic field over a time $T \gg \frac{a}{c}$. What will happen and why?
- b) Find the rate of change of the angular velocity of the cylinder (it has moment of inertia I) per unit length)
- c) Calculate the angular momentum per unit length and show that it is conserved
- d) Discuss the limiting cases of a heavy and light cylinder

- 2) The charge density of a medium with dielectric constant ϵ and conductivity σ is $\rho_0(r) \propto r^2$ at $t=0$ while the total charge is finite
- a) show that the charge decays according to $\rho(r, t) = \rho_0(r) e^{-4\pi t \sigma / \epsilon}$
 - b) Where does the charge go? For the case of a spherical charge distribution show that the outgoing current is equal to the change in charge.
- 3a) show that the continuity of $\vec{E}_{tangential}$ implies the continuity of $\frac{d\vec{B}_{normal}}{dt}$ provided that \vec{E} and \vec{B} satisfy the Maxwell eqs. in both media.
- 3b) Show that the continuity of $\vec{H}_{tangential}$ implies that $\frac{d\vec{B}_{normal}}{dt} = 4\pi \frac{d\rho}{dx}$ provided that the Maxwell eqs. hold in both media inhomogeneous.

g.) A dielectric sphere with radius a and dielectric constant ϵ rotates with angular velocity ω about \hat{x} in a uniform applied electric field E_0 also in the direction of \hat{x} . Show that there is a magnetic field

$$\mathbf{H} = -\vec{\nabla} \Phi$$

$$\Phi = \frac{3}{5} \frac{\epsilon-1}{\epsilon+3} E_0 \frac{\omega a^5}{c r_>^5} xz$$

with $r_>$ the larger of r and a .