

HW network set # 2

1. In this problem we discuss the complex method to solve the Laplace equation.

$$\partial_x^2 + \partial_y^2 \phi = 0 \quad \text{define } \chi \text{ by}$$

$$\partial_x \phi = \partial_y \chi, \quad \partial_y \phi = -\partial_x \chi$$

a) show that the equation for χ can be integrated analogous to solving the potential from the electric field using $\vec{\nabla} \times \vec{E} = 0$

b) Show that $w(x, y) = \phi + i\chi$ is an analytic function of $x + iy$ (Hint: show that it satisfies the Cauchy-Riemann equations)

c) If w is an analytic function that maps a curve C to the line $\text{Re}(w) = \text{constant}$ then $\text{Re}(w)$ satisfies the Laplace equation with boundary condition $\text{Re}(w) = \text{constant}$ on C . Use this to find the potential of a line with constant charge density (Hint: use $w(z) = \log z$)

2) Find the general solution of the 3d Laplace equation in Cartesian coordinates. (You can find this in Jackson). Use an expansion in these solutions to find the potential between two infinite parallel plates at potential $\pm V$ with separation d .



3) Consider a sphere with charge density $\rho = \rho_0 \cos^2 \theta$. The radius of the sphere is R . Use the multipole expansion to find the potential for $r > R$.

4) Use image charge to find the potential of 2 perpendicular plates at $\phi = 0$

with charge q at distance a from one plate and distance b from the other plate.

