

(6)

$$\Rightarrow S = k_B \log\left(\frac{N}{2}\right)$$

Free energy  $F = -NJ + 4J - k_B T \log\left(\frac{N}{2}\right)$

Free energy is lowered  
will be lowered even more by the creation

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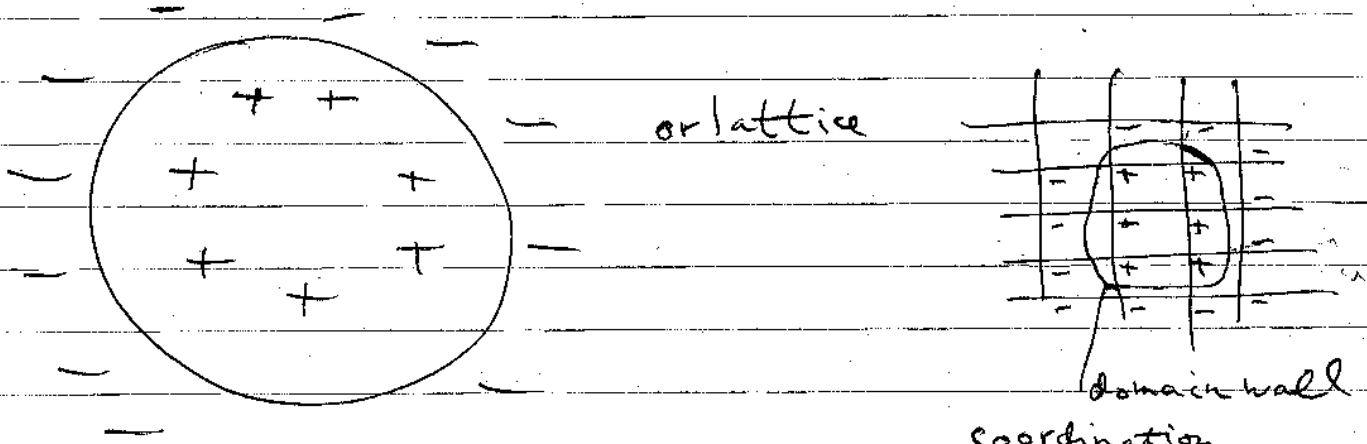
Free energy is lowered  
will be lowered even more by the creation  
of additional domain walls,

$\Rightarrow$  no long range order in 1d  
 $\Rightarrow$  no spontaneous magnetization

(Landau-Peierls argument)

2c) Phases in 2d

Consider a domain wall in 2d



boundary contains  $n$  ~~is~~ bonds

coordination number  $z = 4$

$$E = E_0 + 2nJ$$

We count the # of possibilities to create a domain wall ignoring the fact that it closes on itself.

At each point we have  $z-1$  different possibilities

$$\Rightarrow \Delta S = k_B \log(2-1)^n$$

$$\Delta E = 2nJ$$

$$\Rightarrow \Delta F = n(2J - \log(2-1) k_B T)$$

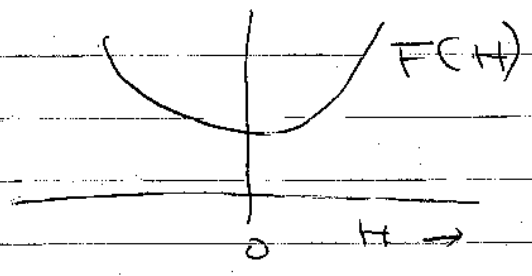
$\Rightarrow$  if  $T > T_c = \frac{2J}{k_B \log(2-1)}$  it pays to create a domain wall.

$T < T_c$  spontaneous magnetization

### 2d) Spontaneous symmetry breaking

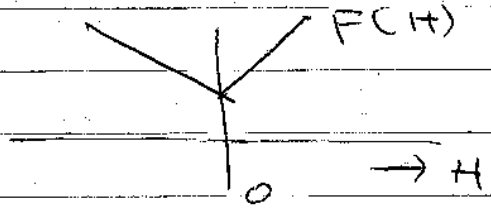
$$Z = \text{Tr} \cdot e^{-\beta \sum_{\langle ij \rangle} S_i S_j - H \sum_i S_i}$$

$\Rightarrow Z(H) = Z(-H)$  and  $F(H) = F(-H)$   
 (just change the sign of the spin in the sum)

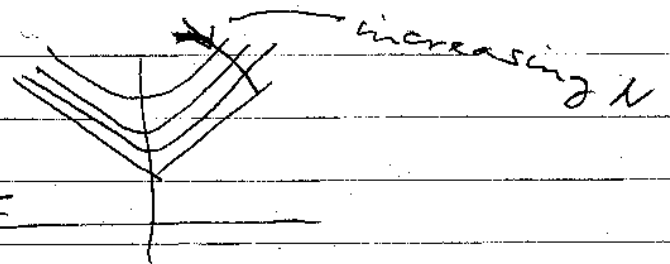


$\Rightarrow M = -\frac{1}{N} \partial_H F = 0$  unless  $F$  has a cusp

i.e.

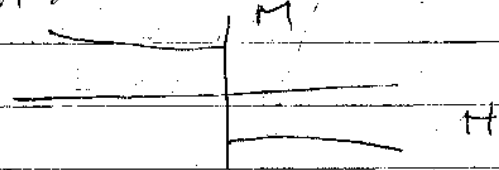


This is not possible for finite  $N$ . A discontinuity may arise for  $N \rightarrow \infty$



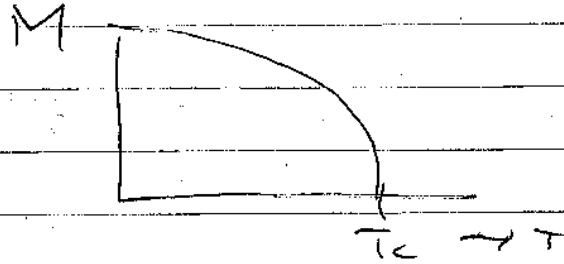
$$M = \lim_{H \rightarrow 0} \lim_{N \rightarrow \infty} -\frac{1}{N} \partial_H F$$

$$\neq \lim_{N \rightarrow \infty} \lim_{H \rightarrow 0} -\frac{1}{N} \partial_H F = 0$$



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A different ensemble of states contributes to the partition function for  $H = 0^+$  and  $H = 0^-$



$T < T_c$  broken phase  
 $SO(2)$  symmetry after averaging

$T > T_c$   $SO(3)$  symmetry after averaging

The broken phase has less symmetry. Therefore we need an additional parameter to describe the system. This parameter is called an order parameter. In the case of the Ising model it is the magnetization

Example of spontaneous symmetry breaking

