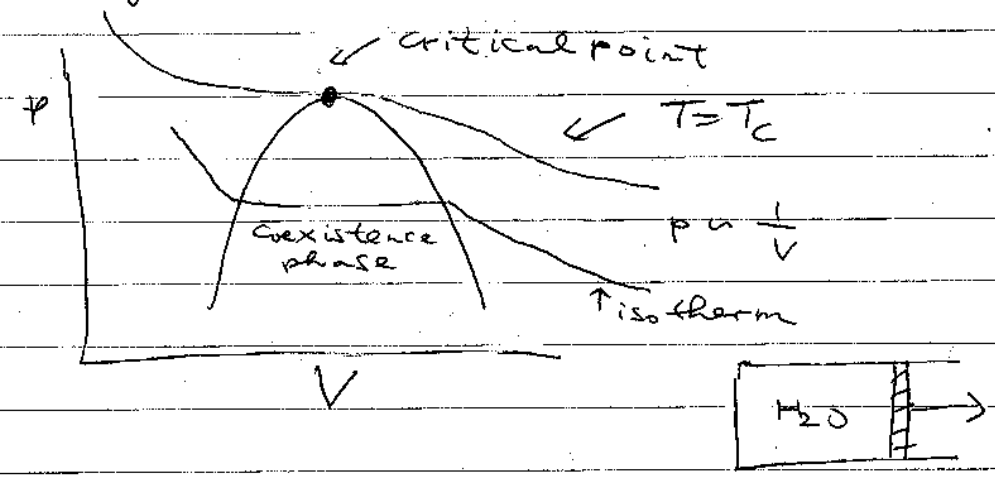
Lecture #1

1a) Examples of phase transitions

i) H_2O



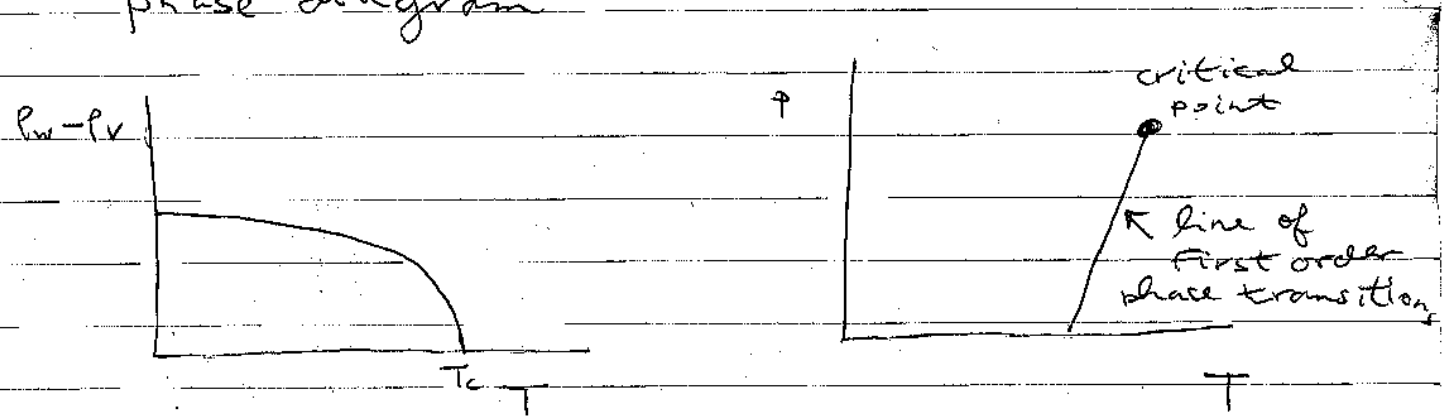
$T < T_c$ $\quad \rho_{water} - \rho_{vapor} > 0$

$T > T_c$ $\quad \rho_{water} = \rho_{vapor}$

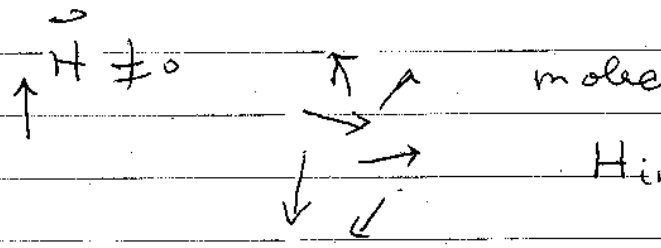
$$\left| \rho_{T>T_c} - \rho_{T<T_c} \right| \sim |(T - T_c)|^{0.327 \pm 0.006}$$

critical exponent

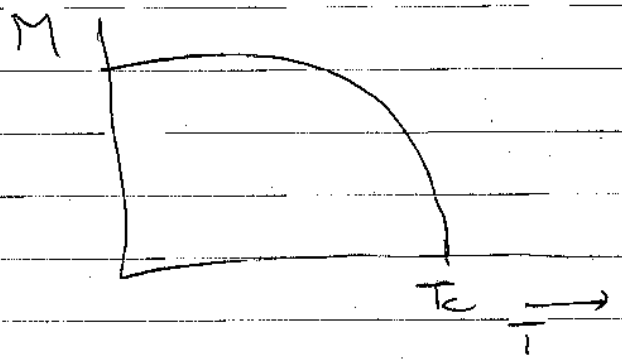
phase diagram



ii) Paramagnetism



magnetization (density of magnetic moment



$M \sim |T_c - T|^{0,311 \pm 0,005}$

the two critical exponents are very close

15) Order of a phase transition

- first order phase transition
  - two phases coexist
  - latent heat ($\frac{1}{V} \partial_\beta F$ is discontinuous)

- 2nd order phase transition
  2nd derivative of the free energy shows a discontinuity
  e.g. $\varepsilon = \frac{1}{V} \partial_\beta F$ is continuous
  $c = \frac{\partial \varepsilon}{\partial T} = \frac{1}{V} T \partial_\beta F$ is discontinuous

phase transitions that are not first order are also called continuous phase transitions

1c) Partition function

$$Z = \mathrm{Tr}\, e^{-\frac{\beta H}{k_B}} \qquad \beta = \frac{1}{T}$$

(k_B: Boltzmann's constant)

$$-\frac{H}{k_B T} = \sum K_n \Theta_n$$

(K_n: coupling constants; Θ_n: local operators)

Free energy $\quad F = -k_B T \log Z$

1d) Thermodynamic limit

$$F = V f_B + S f_S$$

(f_B: bulk free energy density; f_S: surface free energy density)

thermodynamic limit $\quad f_B = \lim_{V\to\infty} \frac{F}{V}$

$$f_S = \lim_{V\to\infty} \frac{F - V f_B}{S}$$

thermodynamic limit does not exist for $1/R^2$ force (Example: gravity)

1e) phase boundaries

parameter space : $K_1, \ldots, K_p$

singular loci : points where $f(K_1, \ldots, K_p)$ is singular

dimension of singularity : D_S
co-dimension $c = d - D_S$

phase boundaries : loci of co-dimension 1

example : water-vapor

1f) Equation of state

$$p = F(T, \rho)$$
$$M = F(T, H)$$

relation between the thermodynamic parameters of a problem

2. Spin models

2a) Examples

i) Ising model $H = -J \sum_{\langle ij \rangle} s_i s_j - H \sum_i s_i$

s_i: either up or down

$\langle ij \rangle$: i and j are neighbors

$$Z = \sum_{S_1=0}^{1} \cdots \sum_{S_N=0}^{1} e^{-\frac{H}{k_B T}}$$

ii) Heisenberg model $H = -J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - \sum_i \vec{H} \cdot \vec{S}_i$

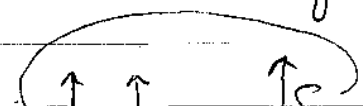
$\vec{S}_i$ can point in any directions

ground state $\uparrow\uparrow \ldots \uparrow$ or $\downarrow\downarrow \ldots \downarrow$
or $\nearrow\nearrow \ldots \nearrow$

for $T \to \infty$ all spins are isotropically distributed in spin space

For the Heisenberg model there is a continuous Family of ground states

2b) Absence of phase transitions in 1d spin systems



$S_{N+1} = S_1$ periodic boundary condition

We look at the Ising model

$E_0 = -NJ$

$S = k_B \log(\#\text{ states}) = 0 \Rightarrow F = -NJ$

↑ Free energy

$E_1 = -NJ + 4J$ $\quad \uparrow\uparrow \underbrace{\downarrow \cdots \downarrow}_{+2J} \underbrace{\uparrow \cdots \uparrow}_{+2J}$

$\binom{N}{2}$ different possibilities for location of spin flip