1 Homework 7, due April 8, 2020

Write down a Monte-Carlo code for the 2d Ising model, preferably using Fortran. The partition function is given by

$$Z = \sum_{\{S_{\vec{i}}\}} e^{K \sum_{\langle \vec{i}\vec{j} \rangle} S_{\vec{i}} S_{\vec{j}} + h \sum_{\vec{i}} S_{\vec{i}}}, \tag{1}$$

where the sum runs over all spins of a two-dimensional lattice and $\langle \vec{i}\vec{j} \rangle$ denotes nearest neighbores Use periodic boundary conditions, $S_{i,N+1} = S_{i,1}$ and $S_{N+1,j} = S_{1,j}$. Update the spins using the Metropolis algorithm, and updates the spins row by row. Do simulations in the range of 50×50 to 100×100 lattices.

a) Calculate the specific heat as a function of the temperature and find the critical temperature. For each temperature, draw a graph of the specific heat versus the sweep number to make sure that the calculations have converged. Compare with the theoretical value of the critical temperature.

b) For $T < T_c$ calculate the magnetization versus h. You can do it for example for $T = \frac{1}{2}T_c$. Again check that the calculations have converged.

c) Do the same for $T = 2T_c$.

d) At $T = T_c$ calculate the magnetization versus h and calculated the critical exponent δ. Make an estimate of the error and compare with the theoretical result. Again check the convergence. Do you see critical slowing down?

Send me also a copy of your program and your results.