## 1 Homework 3, due February 26, 2020

The transfer matrix of the Ising model is given by

$$
\begin{equation*}
V=\left(e^{2 K}-e^{-2 K}\right)^{n / 2} \prod_{q>0} e^{2 K\left(n_{q}+n_{-q}\right) \cos q+K \sin q\left(c_{q} c_{-q}+c_{q}^{*} c_{-q}^{*}\right)+2 \theta\left(n_{q}+n_{-q}-1\right)} \tag{1}
\end{equation*}
$$

with

$$
\begin{equation*}
\cosh \theta=\frac{e^{K}}{\sqrt{e^{2 K}-e^{-2 K}}} \tag{2}
\end{equation*}
$$

a) Show that this can be written as

$$
\begin{equation*}
\left(e^{2 K}-e^{-2 K}\right)^{n / 2} \prod_{q>0} e^{2 K \cos q-\epsilon_{q}\left(n_{q}^{\prime}+n_{-q}^{\prime}-1\right)} \tag{3}
\end{equation*}
$$

with

$$
\begin{equation*}
c_{q}^{\prime}=\cos \phi_{q} c_{q}+\sin \phi_{q} c_{-q}^{*}, \tag{4}
\end{equation*}
$$

and $n_{q}^{\prime}$ the corresponding number operator. The $\epsilon_{q}$ are given by

$$
\begin{equation*}
\cosh \epsilon_{q}=\cosh 2 K \cosh 2 \theta-\cos q \sinh 2 K \sinh 2 \theta \tag{5}
\end{equation*}
$$

b) Show that

$$
\begin{equation*}
\cosh \epsilon_{q}=\frac{\cosh ^{2} 2 K}{\sinh 2 K}-\cos q \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
\epsilon_{q}=\frac{1}{2 \pi} \int_{0}^{2 \pi} d t \log \left(2 \frac{\cosh ^{2} 2 K}{\sinh 2 K}-2 \cos q-2 \cos t\right) \tag{7}
\end{equation*}
$$

