1 Homework 3, due February 26, 2020

The transfer matrix of the Ising model is given by

$$V = (e^{2K} - e^{-2K})^{n/2} \prod_{q>0} e^{2K(n_q+n_{-q})\cos q + K\sin q(c_q c_{-q} + c_q^* c_{-q}^*) + 2\theta(n_q+n_{-q}-1)} \tag{1}$$

with

$$\cosh\theta = \frac{e^K}{\sqrt{e^{2K} - e^{-2K}}}. \tag{2}$$

a) Show that this can be written as

$$(e^{2K} - e^{-2K})^{n/2} \prod_{q>0} e^{2K\cos q - \epsilon_q(n'_q + n'_{-q} - 1)} \tag{3}$$

with

$$c'_q = \cos\phi_q c_q + \sin\phi_q c_{-q}^*, \tag{4}$$

and n'_q the corresponding number operator. The ϵ_q are given by

$$\cosh\epsilon_q = \cosh 2K \cosh 2\theta - \cos q \sinh 2K \sinh 2\theta. \tag{5}$$

b) Show that

$$\cosh\epsilon_q = \frac{\cosh^2 2K}{\sinh 2K} - \cos q, \tag{6}$$

and

$$\epsilon_q = \frac{1}{2\pi}\int_0^{2\pi} dt \log\left(2\frac{\cosh^2 2K}{\sinh 2K} - 2\cos q - 2\cos t\right). \tag{7}$$