1 Homework 1, due February 12, 2020

Consider an infinite range Ising model with Hamiltonian given by

$$H = -\frac{\kappa}{2}\sum_{ij} S_i S_j - H\sum_i S_i, \tag{1}$$

where the sums run over all N spins.
a) Using the Hubbard-Statonovich transformation show that the partition function is given by

$$Z = \int \frac{dx}{\sqrt{\pi/N^2\beta\kappa}} e^{-N\beta L}, \tag{2}$$

where

$$L = \kappa N x^2 - \frac{1}{\beta}\log[2\cosh(\beta H + \beta\kappa N x)]. \tag{3}$$

Why does the large N limit of this model only make sense when κN is kept fixed? Denote κN by κ_0.
b) The integral can be evaluated by a saddle-point approximation. The relevant solution is given by the global minimum. Draw a graph of $L(x)$ for $T < T_c$ and $T > T_c$, first for $H = 0$ and then for a small nonzero value of H.
c) Find an expression for the magnetization in terms of the solutions of the saddlepoing equation, and draw a graph of the magnetization at $H \to 0$ versus the temperature. Find the critical exponent β
d) Find the critical exponent δ,
e) Calculate the susceptibility versus temperature for $H \to 0$ and find the critical exponent γ.