Solutions of HW 9

1. Consider

$$S_N(x,y) = \sum_{k=0}^{N-1} (x-y)\, T_k(x)\, T_k(y)$$

we have recursion $x T_n = b_n T_{n+1} + a_n T_n + b_{n-1} T_{n-1}$

$$S_N(x,y) = (x-y)\, T_{N-1}(x)\, T_{N-1}(y) + S_{N-1}(x,y)$$

$$= \left(b_{N-1} T_N(x) + \cancel{a_{N-1} T_{N-1}(x)} + \cancel{b_{N-2} T_{N-2}(x)}\right) T_{N-1}(y)$$

$$- \; T_{N-1}(x)\left(b_{N-1} T_N(y) + \cancel{a_{N-1} T_{N-1}(y)} + \cancel{b_{N-2} T_{N-2}(y)}\right)$$

$$+ \; b'_{N-2}\left(\cancel{T_{N-1}(x) T_{N-2}(y)} - \cancel{T_{N-2}(x) T_{N-1}(y)}\right)$$

if $b'_{N-2} = b_{N-2}$

$$= b_{N-1}\left(T_N(x)\, T_{N-1}(y) - T_{N-1}(x)\, T_N(y)\right)$$

2) 2-12 of GS

a) $$\frac{\pi(\xi_\mu)}{\xi_\mu - \xi_\nu} = 0 \text{ for } \mu \neq \nu$$

$$= \pi'(\xi_\mu) \text{ for } \mu = \nu$$

b) Polynomials ord N-1 are defined uniquely by N parameters which can be zero

c) $$f_{2N-1}(x) = P_N(x) \underbrace{Q(x)}_{\text{order } N-1} + \underbrace{R(x)}_{\text{order } N-1}$$

just division of f_{2N-1} by P_N

d) $$f_{2N-1}(x_\nu) = \underbrace{P_N(x_\nu)}_{=0} Q(x_\nu) + R(x_\nu)$$

e) $$\int \overset{w(x)}{F(x)}\, dx \underset{\text{orthogonality}}{=} \int_a^b w(x)\, R(x)$$

use $P_N(x)$ for $\pi(x)$ in a

$$= \int_a^b f_{2N-1}(x_\nu) \frac{P_N(x)}{P_N'(x_\nu)(x - x_\nu)} = \sum_{\nu=1}^{N} w_\nu f_{2N-1}(x_\nu)$$

3.

```
:= FullSimplify[D[D[HermiteH[n, x] * Exp[-x ^ 2 / 2], x], x],
    x * HermiteH[-1 + n, x] == 1 / 2 * HermiteH[n, x]  + (n - 1) HermiteH[n - 2, x]]
```

]= $e^{-\frac{x^2}{2}} \left(-1 - 2 n + x^2\right)$ HermiteH[n, x]