Solution of HW 8

1, Do this yourself by Mathematica

2, $P_0 = 1$, $P_1 = x$, $P_2 = x^2 + a_2$

$P_3 = x^3 + a_3 x$, $P_4 = x^4 + a_4 x^2 + b_4$

$P_5 = x^5 + a_5 x^3 + b_5 x$

because the weigh function is even.

These polynomials are monic and normalized to $\int P_n^2(x)\, w(x) = h_n \neq 1$

Even and odd polynomials are always orthogonal

$$(P_2 \cdot P_0) = \int_{-1}^{1} (1+x^2)(x^2+a_2) = \underbrace{2a_2 + (a_2+1)\frac{2}{3} + \frac{2}{5}}_{=0}$$

$$\Rightarrow \frac{8}{3} a_2 = -\frac{16}{5} \Rightarrow a_2 = -\frac{3}{5}$$

$$(P_3 \cdot P_1) = \int_{-1}^{1} (1+x^2)(x^3+a_3 x)\, x\, dx$$

$$= \int_{-1}^{1} dx \left(a_3 x^2 + x^4(a_3+1) + x^6\right)$$

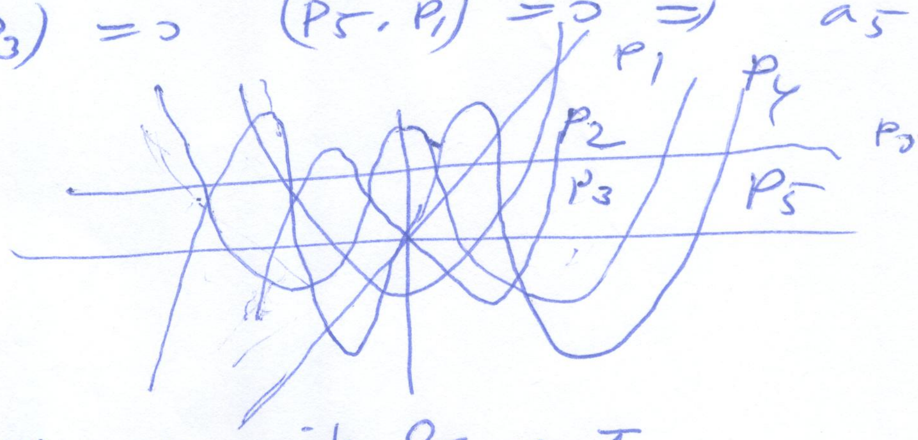
$$= \frac{2}{3} a_3 + \frac{4 \cdot 2}{5}(a_3+1) + \frac{2}{7} = 0$$

$$\Rightarrow \frac{34}{15} a_3 = -\frac{66}{7} \Rightarrow a_3 = -\frac{15}{34} \cdot \frac{66}{7}$$

$(P_4 \cdot P_2) = 0$ $\quad (P_4 \cdot P_0) = 0 \Rightarrow a_4 = \quad , b_4 =$

$(P_5 \cdot P_3) = 0$ $\quad (P_5 \cdot P_1) = 0 \Rightarrow a_5 = \quad b_5 = ,$

b)



c) zeros are interlacent

2.3 of GS

$$F(s,t) = \int_{-\infty}^{\infty} e^{-x^2-2sx-s^2-2tx-t^2} dx$$

$$= \int_{-\infty}^{\infty} e^{-(x-s-t)^2+(s+t)^2-s^2-t^2}$$

$$= \sqrt{\pi}\, e^{2st}$$

only equal powers of s and t

$$\Rightarrow \int H_n H_m e^{-x^2} dx = 0$$

$$F(s,t) = \int_{-\infty}^{\infty} e^{-x^2} \sum_{n=0}^{\infty} \frac{1}{n!} t^n H_n(x) \sum_{m=0}^{\infty} \frac{1}{m!} t^m H_m(x)$$

power of $(st)^n$

$$\int_{-\infty}^{\infty} e^{-x^2} \frac{H_n(x) H_n(x)}{n!\, n!} = \sqrt{\pi}\, \frac{2^n}{n!}$$

$$\Rightarrow \int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_n(x) = \sqrt{\pi}\, n!\, 2^n$$