<u>HW3</u>

1) $\lambda e_1 + \mu e_2 = 0 \Rightarrow \begin{array}{l}\lambda + \mu = 0\\ 2\lambda - i\mu = 0\end{array}$

$\Rightarrow (2+i)\mu = 0$

$\Rightarrow \mu = 0 \Rightarrow \lambda = 0$

$g_{\mu\nu} = \begin{pmatrix} 5 & 1-2i \\ 1+2i & 2 \end{pmatrix}$

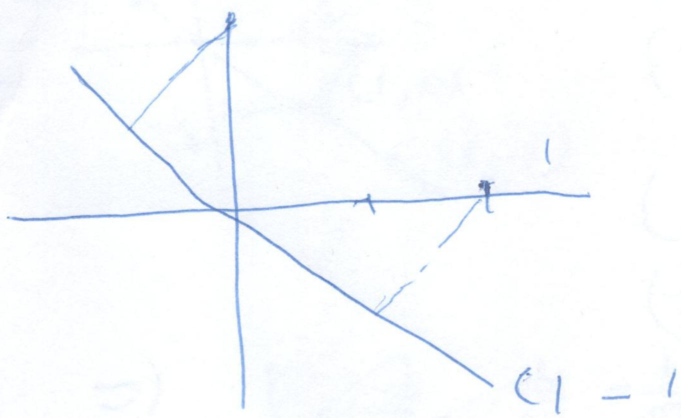
b) $\begin{pmatrix} i \\ 3 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -i \end{pmatrix} \Rightarrow \begin{array}{l}\lambda + \mu = i\\ 2\lambda - i\mu = 3\end{array}$

$\Rightarrow (2+i)\mu = 2i-3$

$\Rightarrow \lambda = \frac{2i-3}{2+i}$

$\lambda = i - \mu$

3)



$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$ $\quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$

$\Rightarrow P = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad P^2 = P$

Ad

$$P_1^2 = P_1$$

$$P_2^2 = (1-P_1)^2 = 1 - 2P_1 + P_1^2 = 1 - P_1$$

$\Rightarrow$ $1-P_1$ is a projection operator

$$P_1 P_2 = P_1(1-P_1) = P_1 - P_1^2 = P_1 - P_1 = 0$$

$\operatorname{Im} P_2 = \ker P_1$

if $x \in \ker P_1 \Rightarrow P_1 x = 0$

$$P_2 x = (1-P_1)x = x$$

$\Rightarrow x \in \operatorname{Im} P_2$

if $y \in \operatorname{Im} P_2 \Rightarrow \exists x \mid y = P_2 x$

$$P_1 y = P_1 P_2 x = P_1((1-P_1)x)$$

$$(P_1 - \underset{P_1}{\underset{\parallel}{P_1^2}})x = 0$$

$\ker P_2 = \operatorname{Im} P_1$; same proof

$P_2 \to 1 - P_1$

$P_1 \to 1 - P_2$