Solutions of HW 2

A.1 i) $x+\hat{0}=x$, $x+0=x$ } subtrat $\hat{0}-0=0$

$\Rightarrow \hat{0}=0$

ii) $0x+x=(0+1)x=x$

$\Rightarrow 0x=\vec{0}$

iii) $x+y=0$, $-x=-x$ } $\Rightarrow y=-x$

iv) $x+z_1=y$, $x+z_2=y$ } subtract $\Rightarrow z_1=z_2$

$z_1=y-x$

v) $\lambda 0=0$

$\lambda 0+x$

$=\lambda(x-x)+x$

$=x(\lambda-\lambda+1)$

$=x(0+1)=x$

$\Rightarrow \lambda 0=0$

vi) $\lambda x=0$ $\lambda\neq 0$ $\Rightarrow \lambda^{-1}\lambda x=\lambda^{-1}0=0$

$\Rightarrow x=0$

$x\neq 0$ suppose $\lambda\neq 0$ $\lambda x=0$

multiply by λ^{-1} $\Rightarrow x=0$ $\lightning$

contradicts assumption $\Rightarrow \lambda=0$

vii) $(-1)x+x=(-1)x+(1)x=(-1+1)x=0x=0$

see ii)

$\Rightarrow (-1)x=-x$

A5) $A e_\mu = A_{\mu\nu} e_\nu$

$A^*(f)(x) = f(Ax)$

e_ν^* is dual basis $\quad e_\nu^* e_\mu = \delta_{\nu\mu}$

$$A^*(e_\nu^*) e_\mu = e_\nu^* A_{\mu\nu'} e_{\nu'}$$
$$= \delta_{\nu\nu'} A_{\mu\nu'} \quad = A_{\mu\nu}$$
$$= A_{\mu\nu} e_\mu^*(e_\mu)$$
$$\Rightarrow A^*(e_\nu^*) = A_{\mu\nu} e_\mu^*$$
$$= A^T_{\nu\mu} e_\mu^*$$

A6) $\langle y, Ax\rangle = \langle A^+ y, x\rangle$

$$(y, ABx = \langle (AB)^+ y, x$$
$$= (A^+ y, Bx) = (B^+ A^+ y, x)$$
$$\Rightarrow (AB)^+ = B^+ A^+$$

A d

$P_1^2 = P_1$

$P_2^2 = (1-P_1)^2 = 1 - 2P_1 + P_1^2 = 1 - P_1$

$\Rightarrow$ $1-P_1$ is a projection operator

$P_1 P_2 = P_1(1-P_1) = P_1 - P_1^2 = P_1 - P_1 = 0$

$\mathrm{Im}\, P_2 = \ker P_1$

if $x \in \ker P_1 \Rightarrow P_1 x = 0$

$P_2 x = (1-P_1)x = x$

$\Rightarrow x \in \mathrm{Im}\, P_2$

if $y \in \mathrm{Im}\, P_2 \Rightarrow \exists x \mid y = P_2 x$

$P_1 y = P_1 P_2 x = P_1((1-P_1)x)$

$(P_1 - \underset{P_1}{\underset{\shortparallel}{P_1^2}})x = 0$

$\ker P_2 = \mathrm{Im}\, P_1$; same proof

$P_2 \to 1 - P_1$

$P_1 \to 1 - P_2$