Homework Set 8. Due Friday October 21 at 10.30 am

1. This problem can worked out using Mathematica. Consider the orthogonal polynomias

$$P_n(x) = \sqrt{2}\sin(\pi n x) \tag{1}$$

with inner product

$$(f,g) = \int_0^1 f(x)g(x)ds. \tag{2}$$

   a) Consider the expansion $\sqrt{\frac{1}{4}-(x-\frac{1}{2})^2} = \sum_{k=0}^N a_k P_k(x)$ and calculate the first 100 a_k. Why are the even coefficients equal to zero?

   b) Draw a graph of $\sum_{k=0}^N a_k P_k(x)$ for $N = 10$, 20, 100 and compare to the graph of $\sqrt{\frac{1}{4}-(x-\frac{1}{2})^2}$.

   c) . Calculate the L_2 norm of the difference between $\sqrt{\frac{1}{4}-(x-\frac{1}{2})^2}$ and the approximation for $N = 10$, 20, 30, 40, 50, 60, 80 , 90, 100 and draw a graph of the L_2 norm versus $1/N$ and $1/N^2$. What is your conclusion?

2. Consider the inner product

$$(f,g) = \int_{-1}^1 x(x)f(x)g(x) \qquad \text{with} \qquad w(x) = 1 + x^2. \tag{3}$$

   a) Construct the first five orthogonal polynomials.

   b) Plot them in one figure (you can also use Mathematica to do this).

   c) What can you say about the zeros of these polynomials?

3. Do Excercise 2.3 of Goldbart and Stone, p. 64.