Homework Set 5. Due Friday September 30 at 10.30 am

1. If $\gamma_5 = (\underbrace{1,\cdots 1}_{m}, \underbrace{-1,\cdots,-1}_{n}$ and

$$H\gamma_5 + \gamma_5 H = 0, \tag{1}$$

show that the Hermtian matrix H is of the form

$$H = \begin{pmatrix} 0 & C \\ C^\dagger & 0 \end{pmatrix} \tag{2}$$

with C a $m \times n$ matrix. Also show that the nonzero eigenvalues occur in pairs $\pm\lambda$ and relate the eigenvectors corresponding to this pair of eigenvalues.

2. Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix} \tag{3}$$

a) Calculate $\det A$.
b) Calculate the adjugate matrix of A.
c) Use this to find A^{-1} and check that $AA^{-1} = 1$.

3. This is a numerical problem. Using Mathematica construct an ensemble of 1000 100×100 Hermitian random matrices with probability distribution

$$P(H) = ce^{-N\mathrm{Tr}H^2}, \qquad N = 100. \tag{4}$$

a) Make a histogram of the eigenvaliues.
b) Show that the histogram is fitted by a $\sqrt{R^2 - x^2}$. What is the value of R? c) What is the area below the curve? Interpret the number.