

Homework Set 4. Due Friday September 23 at 10.30 am

1. Consider the matrix

$$\begin{pmatrix} a & b \\ \lambda a & \lambda b \end{pmatrix} \quad (1)$$

- a) Calculate $\ker(A)$.
- b) Calculate $\text{Im}(A)$.
- c) Calculate $\ker(A)^\perp$.
- d) Is $\mathcal{R}_2 = \ker(A) + \text{Im}(A)$? Is $\mathcal{R}_2 = \ker(A) + \ker(A)^\perp$? Is $\text{Im}(A) = \ker(A)^\perp$?

2. Show that for $n \times n$ matrices a, b, c and d

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \det d \det(a - bd^{-1}c) \quad (2)$$

3. Consider the matrix

$$H = \begin{pmatrix} 0_m & C \\ C^\dagger & 0_n \end{pmatrix} \quad (3)$$

with C a $m \times n$ matrix and 0_k a $k \times k$ matrix of zeros.

- a) Calculate the kernel of H . Hint: consider first the case $m = 2$ and $n = 1$.
- b) What is the rank of H ?