Homework Set 10. Due Friday November 11 at 10.30 am

1. Show that by acting on a test function that

$$\lim_{\epsilon \to 0} \frac{1}{x + i\epsilon} = P\left(\frac{1}{x}\right) - \pi i \delta(x). \tag{1}$$

2. Do (3.1) of Goldbart and Stone, p. 98.

3. Consider the differential equation

$$-\frac{d^2 y}{dx^2} + (4x^2 - 2)y = 0. \tag{2}$$

a) Show that $y = e^{-x^2}$ is a solution of this equation.

b) Write down the differential equation for the Wronskian and show that the Wronskian is constant.

c) If y_2 is a second solution of the differential equation, use the Wronskian to derive a first order equation for y_2.

d) Show that the solution for y_2 is given by

$$y_2 = e^{-x^2} \int_{x_0}^{x} e^{2y^2} dy. \tag{3}$$

e) Why does this solution not play a role in quantum mechanics?