

- 1) Consider the inner product $(f, g) = \int_{-1}^1 w(x) f(x) g(x) dx$
 with $w(x) = 1+x^2$.
- Construct the first 5 orthogonal polynomials
 - Plot them in one figure
 - What can you say about the zeros of these polynomials
- 2) Do Exercise 2.12 of GS.
- 3) Show that the Tchebychev polynomials T_n satisfy the Christoffel-Darboux formula
- $$\sum_{k=0}^{N-1} T_k(x) T_k(y) = \frac{b_{N-1} (T_N(x) T_{N-1}(y) - T_{N-1}(x) T_N(y))}{x - y}$$
- 4) Show that by acting on a test function that
- $$\lim_{\varepsilon \rightarrow 0} \frac{1}{x + i\varepsilon} = P\left(\frac{1}{x}\right) - \pi i \delta(x)$$