

1. Consider the matrix

$$H = \begin{pmatrix} \Theta_m & C \\ C^\dagger & \Theta_n \end{pmatrix} \quad (**)$$

with C an $m \times n$ matrix and Θ_m is an $m \times m$ block of zeros.

a) Show that generically H has $m-n$ zero eigenvalues.

b) Find the eigenvectors of the zero modes

Hint: Cook First at $m=2, n=1$

2. Show that the outer product of two vector \vec{a} and \vec{b} in 3d is invariant under rotations.

b) Use this to show that $|\vec{a} \times \vec{b}|$ is equal to the area spanned by \vec{a} and \vec{b}

3. Show that $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \det d \det(a - b d^{-1} c)$

4) If $\gamma_5 = \underbrace{(1, \dots, 1)}_m, \underbrace{(-1, \dots, -1)}_n$

and $H \gamma_5 + \gamma_5 H = 0$, show that the Hamiltonian has the form (**).

Also show that the nonzero eigenvalues occur in pairs $\pm \lambda$ and relate the eigenvectors.