

Homework set #5, due September 30, 2020

1) Consider the inner product  $(f, g) = \int_{-1}^1 w(x) f(x) g(x) dx$   
with  $w(x) = 1+x^2$ .

- Construct the first 5 orthogonal polynomials
- Plot them in one figure
- What can you say about the zeros of these polynomials

2) Do Exercise 2.12 of GS.

3) Show that the Chebyshev polynomials  $T_n$  satisfy the Christoffel-Darboux formula

$$\sum_{k=0}^{N-1} T_k(x) T_k(y) = \frac{b_{N-1} (T_N(x) T_{N-1}(y) - T_{N-1}(x) T_N(y))}{x - y}$$

4) Show that by acting on a test function that

$$\lim_{\varepsilon \rightarrow 0} \frac{1}{x + i\varepsilon} = P\left(\frac{1}{x}\right) - \pi i \delta(x)$$