

Homework #4, due September 23, 2020

1. Consider the symplectic form

$$x_1 x_2 - 2 x_1 x_3 - 2 x_2 x_4 - 4 x_2 x_3 - 4 x_3 x_4$$

Bring this symplectic form in standard form and determine the dual basis for this form

2) Consider the matrix $A = \begin{pmatrix} 9 & -3 & -5 \\ -3 & 11 & 0 \\ 2 & 6 & 20 \end{pmatrix}$

a) Try to diagonalize this matrix by mathematica and find the eigenvalue and eigenvectors. What is your interpretation.

b) Is this matrix diagonalizable? Find the minimum order polynomial satisfied by this matrix

c) Bring this matrix in the Jordan canonical form.

3) Consider the sequence of functions

$$f_n(x) = x + \frac{x^n}{n} \text{ on } \mathbb{R}$$

a) Does this sequence converge uniformly to $f(x) = x$?

b) If we restrict the domain of the function to $[-2, 2]$, show that the convergence is uniform

c) Is this sequence a Cauchy sequence? Explain why!

4) If $f_n(x) \rightarrow f(x)$ uniformly on $[a, b]$, show that you can interchange the limit and integral