Homework #2 due September 9, 2020

1. Consider the matrix

$$H = \begin{pmatrix} 0_m & C \\ C^{+} & 0_n \end{pmatrix} \quad (**)$$

with C an $m \times n$ matrix and 0_m is an $m \times m$ block of zeros.

a) Show that generically H has $m-n$ zero eigenvalues.

b) Find the eigenvectors of the zero modes

Hint: Look first at $m=2$, $n=1$

2. Show that the outer product of two vector $\vec{a}$ and $\vec{b}$ in 3d is invariant under rotations.

b) Use this to show that $|\vec{a} \times \vec{b}|$ is equal to the area spanned by $\vec{a}$ and $\vec{b}$

3. Show that

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \det d \det(a - b d^{-1} c)$$

4) If $\gamma_5 = (\underbrace{1, \ldots, 1}_{m}, \underbrace{-1, \ldots -1}_{n})$

and $H\gamma_5 + \gamma_5 H = 0$, show that the Hamiltonian has the form $(**)$.

Also show that the nonzero eigenvalues occur in pairs $\pm \lambda$ and relate the eigenvectors.

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