Homework set # 11

1) Show that $U = \begin{pmatrix} \cos\theta\, e^{i\varphi} & \sin\theta\, e^{i\omega} \\ -\sin\theta\, e^{-i\omega} & \cos\theta\, e^{-i\varphi} \end{pmatrix}$

is an SU(2) matrix

b) Calculate $U^{-1} dU \equiv \delta U$

c) Calculate the Jacobian from the variables δU to θ, φ, ω?
First show that $(U^{-1}dU)^{+} = -U^{-1}dU$ and $\mathrm{Tr}\, U^{-1}dU = 0$.
Then calculate the Jacobian from $(\delta U)_{12}(\delta U)_{21}(\delta U)_{11}$ to θ, φ, ω.

2) Show that

$$e^{X}\, e^{Y} = e^{X+Y+\frac{1}{2}[X,Y]+\cdots}$$

with X and Y $n \times n$ matrices and the dots represent term of higher order in X or Y

3) Show that a rotation in an odd-dimensional space always has a rotation axis, i.e. $\exists$ vector $\vec{v}$ such that $O\vec{v} = \vec{v}$.